

Virtual Backbone Construction in MANETs using Adjustable Transmission Ranges *

Jie Wu

Department of Computer Science and Engineering
Florida Atlantic University
Boca Raton, FL 33431
Email: jie@cse.fau.edu

Fei Dai

Department of Electrical and Computer Engineering
North Dakota State University
Fargo, ND 58105
Email: fei.dai@ndsu.edu

Abstract

Recently, the use of a virtual backbone in various applications in mobile ad hoc networks (MANETs) has become popular. These applications include topology management, point and area coverage, and routing protocol design. In a MANET, one challenging issue is to construct a virtual backbone in a distributed and localized way while balancing several conflicting objectives: small approximation ratio, fast convergence, and low computation cost. Many existing distributed and localized algorithms select a virtual backbone without resorting to global or geographical information. However, these algorithms incur a high computation cost in a dense network. In this paper, we propose a distributed solution based on reducing the density of the network using two mechanisms: clustering and adjustable transmission range. By using adjustable transmission range, we also achieve another objective, energy-efficient design, as a by-product. As an application, we show an efficient broadcast scheme where nodes (and only

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nodes) in a virtual backbone are used to forward the broadcast message. The virtual backbone is constructed using Wu and Li's marking process [37] and the proposed density reduction process. The application of the density reduction process to other localized algorithms is also discussed. The efficiency of our approach is confirmed through both analytical and simulation study.

keywords: Adjustable transmission range, broadcasting, clustering, connected dominating set (CDS), energy efficiency, mobile ad hoc networks (MANETs).

1 Introduction

Although a mobile ad hoc network (or simply MANET) has no physical backbone infrastructure, a *virtual backbone* can be formed by nodes in a *connected dominating set* (CDS) of the unit-disk graph of a given MANET. Recently, the use of a virtual backbone in various applications in MANETs has become popular. These applications include topology management in MANETs, point and area coverage in sensor networks, and routing protocol design. A dominating set (DS) is a subset of nodes in the network where every node is either in the subset or a neighbor of a node in the subset. In a unit-disk graph, node connections are determined by their geographical distances. It has been proved that finding the minimum CDS in a unit-disk graph is NP-complete.

A common source of overhead in a MANET comes from blind flooding/broadcasting, where a broadcast message is forwarded by every node in the network exactly once. Broadcasting is used by the route discovery process in several reactive routing protocols. Due to the broadcast nature of wireless communication (i.e., when a source sends a message, all of its neighbors will hear it), blind flooding/broadcasting may generate excessive redundant transmission. Redundant transmission may cause a serious problem, referred to as the broadcast storm problem [31], in which redundant messages cause communication contention and collision. In Figure 1(a), when each node forwards the message once, node w will receive the same message six times. To reduce redundant transmission, nodes (and only nodes) in the virtual backbone forward the broadcast message once when they receive the message for the first time.

In a MANET, one challenging issue is to construct a virtual backbone in a distributed and localized way while balancing several conflicting objectives: small approximation ratio, fast con-

vergence, and low computation cost. Many existing distributed and localized algorithms can select a virtual backbone without resorting to global or geographical information. For example, in Wu and Li's marking process [37], each node is marked (i.e., in a CDS) if it has two unconnected neighbors. The marking process is effective in reducing the size of the CDS. In addition, it supports localized maintenance in a mobile environment. However, the process incurs a high communication and computation cost in a dense network, since each node needs to exchange neighbor sets among 1-hop neighbors and to check all pairs of its neighbors.

In this paper, we propose a distributed solution to reduce the network density before applying a localized CDS algorithm. This method merges two mechanisms: clustering and adjustable transmission range. The basic idea is to first reduce the network density through clustering using a short transmission range. Neighboring clusterheads (i.e., clusterheads that are 2 or 3 hops away) are connected using a long (and normal) transmission range. In this way, neighboring clusterheads are connected without using any gateway selection process. Connected clusterheads form a CDS. Depending on the selection of the short and long transmission ranges, two versions of the distributed solution are given. Then, a localized CDS algorithm is applied on the connected clusterhead set to select a final and smaller CDS.

The objective of our work is to combine the strength of clustering and localized CDS solutions. The clustering scheme constructs a CDS with a constant approximation ratio and a derived graph with bounded node degree. The local scheme, applied to the derived graph, has constant message and time cost and is very effective in reducing the final CDS size for average cases. Several schemes exist that connect clusterheads to form a CDS [3, 4, 11, 23], but these schemes have either relatively high redundancy [3, 23] or high overhead [4, 11]. Therefore, a low-cost scheme to form a small CDS is still desirable.

As an application, we show an efficient broadcasting where the virtual backbone is constructed using the clustering approach, followed by pruning on the clusterhead set with Wu and Li's marking process. The density reduction approach can be used in other localized solutions such as multipoint relay (MPR) [26]. We further extend the distributed solution to a multi-stage density reduction process for very dense networks. In the multi-stage extension, node behaviors in the clustering process vary, depending on local node density. Each node selects a best strategy to minimize the number of clusterheads while maintaining global connectivity. This scheme adapts well to large scale networks with non-uniform node distributions.

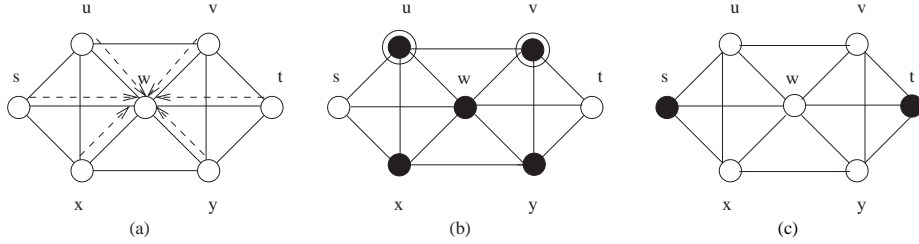


Figure 1: (a) Broadcast storm problem. (b) Marked nodes: black (marked by the marking process) and double circled (survivors after applying Rule k). (c) Clustering approach: black nodes (clusterheads) and white nodes (non-clusterheads).

By using adjustable transmission range, we also achieve several other goals as by-products: reducing the computation complexity of the broadcast algorithm, maximizing the traffic capacity of the network, reducing the power consumption of the broadcast process, prolonging the life span of each individual node, and reducing the contention at the MAC layer.

2 Related work

Wu and Lou [38] gave a comprehensive classification of CDS construction algorithms in MANETs: *global*, *quasi-global*, *quasi-local*, and *local*. Global solutions, such as Guha and Khuller's greedy algorithm [14], are based on global state information and are expensive in MANETs. Quasi-global solutions, such as Alzoubi et al's tree-based approach [4], require network-wide coordination, which causes slow convergence in large scale networks. Many cluster-based approaches [3, 23, 38] are quasi-local. The status (clusterhead/non-clusterhead) of each node depends on the status of its neighbors, which in turn depends on the status of neighbors' neighbors and so on. The propagation of status information is relatively short ($O(\log n)$) on average, but, in the worst case, can span the entire network. Dubhashi et al [11] proposed another quasi-local approach, with bounded ($O(\log n)$) steps of status propagation. In local approaches (i.e., localized algorithms), the status of each node depends on its k -hop information only with a small k , and there is no propagation of status information. Local CDS formation algorithms include Wu and Li's marking process (MP) [37], several MP variations [8, 10], Qayyum, Viennot, and Laouiti's multipoint relay (MPR) [26], and MPR extensions [1, 22, 24], which will be discussed in Section 3.1.

There are two categories of clustering approaches. In *cluster formation approaches* [16, 23], the set of clusterheads is a maximal independent set (MIS), where two clusterheads cannot be neighbors. In unit disk graphs, an MIS is an $O(1)$ approximation of the minimal DS. The set of clusterheads can be used to construct a CDS with an $O(1)$ approximation ratio [3, 4, 11, 23], as will be discussed in Section 3.2. The major drawback of a cluster formation approach is its relatively slow convergency, which takes $O(n)$ rounds in the worst case. In *DS formation approaches* [13, 15, 18, 28], the set of clusterheads may not be a MIS. The best DS formation algorithm takes $O(1)$ rounds, but the DS size is unbounded in the worst case. For unit disk graphs with a uniform node distribution, Gao et al [13] proposed the following local algorithm. Each node selects a node with the highest priority in its neighborhood (including itself) as a clusterhead. The resultant set of clusterheads has an expected $O(\sqrt{n})$ approximation ratio. An iterative application of this algorithm can achieve an expected $O(1)$ approximation ratio in $O(\log \log n)$ rounds. A similar scheme was used by the CEDAR protocol [28] to select a set of *cores* (i.e., dominating nodes). For a general graph, Jia et al [15] proposed a randomized algorithm to compute a DS, which finishes in $O(\log n \log \Delta)$ rounds with high probability, where Δ is the maximal node degree, and has an expected $O(\log n)$ approximation ratio. Kuhn and Wattenhofer [18] proposed another randomized algorithm that achieves an expected $O(k\Delta^{2/k} \log \Delta)$ approximation ratio in $O(k^2)$ rounds, where k is a constant. Kuhn et al [17] proved that no clustering approach can achieve a constant approximation ratio in constant rounds.

The formation of a CDS is sometimes tied with a broadcast process. Wu and Dai [36] classified broadcast algorithms that form a CDS using local solutions as *self-pruning* and *neighbor-designating* methods. In self-pruning methods [8, 10, 25, 29, 30, 37], each node makes its local decision on its status: forwarding (i.e., within the CDS) or non-forwarding (i.e., outside the CDS). In neighbor-designating methods [22, 24, 26], the status of each node is determined by its neighbors. Local methods also have the following two orthogonal classifications based on the way the CDS is constructed: static (before the broadcast process) vs. dynamic (during the broadcast process), and source-independent (independent of the location of the source) vs. source-dependent (dependent on the location of the source). In general, dynamic is better than static in terms of generating a small CDS. Similarly, source-dependent edges out source-independent. However, neither dynamic nor source-dependent methods produce a general purpose CDS – a new CDS is constructed for each source and/or broadcast process.

Several protocols have been proposed to manage energy consumption by adjusting transmis-

sion ranges. In the source-dependent approach (called minimum energy broadcast), the source is given, but the problem is still NP-complete [9]. Clementi et al [9] proved that the minimum energy broadcast problem is approximable with a constant factor in wireless networks. Wieselthier et al [35] proposed several global algorithms. Two of those algorithms, MST (minimal spanning tree) and BIP (broadcasting incremental power), were shown by Wan et al [32] to have a small approximation ratio of 12. Recently, a localized scheme [7] was proposed using a graph density reduction method based on RNG (relative neighborhood graph). This approach uses location information in addition to neighborhood information, which increases the cost.

In the source-independent approach (called topology control), all nodes can be a source and are able to reach all other nodes by assigning appropriate ranges. The problem of minimizing the total transmission power consumption (based on an assigned model) is NP-complete. Several localized solutions exist based on local spanning subgraphs, such as SPT [27], RNG [7], and MST [20]. Recently, new algorithms have been proposed to achieve multiple desirable properties such as low message cost, constant stretch ratio [34], low weight [21], and minimal interference [6]. Another concern is the overhead. Most localized topology control schemes require 1-hop location information, which becomes expensive to collect in very dense networks. An expanding search region mechanism [5, 19, 27] was devised to solve this problem. The cone-based scheme [19] requires only the AoA (angle-of-arrival) information of a few neighbors in a small search region. Probabilistic schemes, such as K-Neigh [5], preserve connectivity with high probability and collect only topology information in the search region. Topology control schemes sparsify a network by removing edges and reducing transmission ranges. Some of them [20, 21, 34] guarantee a bounded node degree. On the other hand, the purpose of CDS construction is to reduce the number of active nodes. Although both approaches conserve energy and bandwidth consumption, they have different sets of applications and cannot replace each other.

In this paper, we use the static and source-independent approach for CDS construction since it is more generic. The resultant CDS is suitable for all situations. We also assume that no location information is provided.

3 Preliminaries

3.1 CDS formation algorithms

Wu and Li [37] proposed a self-pruning process, called *marking process*, to construct a CDS.

Marking process: *Each node is marked if it has two unconnected neighbors; otherwise, it is unmarked.*

The marked nodes form a CDS, which can be further reduced by applying Dai and Wu’s pruning rule k [10] (i.e., changing a marked node back to an unmarked node).

Pruning Rule k : *A marked node can unmark itself if its neighbor set is covered by a set of connected nodes with higher priorities.*

A set U is said to be *covered* by V (and V is called a *coverage set* of U) if every node in U is either in V or a neighbor of a node in V . The node priority can be defined based on node degree (which is dynamic) and/or node ID (which is static). When the coverage set is restricted to a subset of the neighbor set, the corresponding rule is called a *restricted rule*. Dai and Wu have shown that a restricted rule is almost as efficient as the non-restricted rule in reducing the size of the CDS. In the subsequent discussion, we use Rule k to refer to the restricted pruning Rule k . It has been shown that the both marking process (MP) and Rule k require 2-hop information, $O(\Delta)$ message cost, and $O(\Delta^2)$ computation cost, where Δ is the maximal node degree in the network. To apply MP and Rule k , each node needs to check $O(\Delta^2)$ pairs of neighbors, which is costly in dense networks.

Figure 1(b) shows an example of MP and Rule k with node ID as the priority; that is, the lower the ID of a node, the higher the priority of the node (e.g., u has a higher priority than w). Nodes u , v , w , x , and y are marked after applying MP. Nodes x and y are unmarked by Rule k , since their neighbor sets are covered by w . Node w is also unmarked by Rule k , since its neighbor set is jointly covered by u and v , which are directly connected.

3.2 Clustering approach

The clustering approach is commonly used to offer scalability and is efficient in a dense network. Basically, the network is partitioned into a set of clusters, with one clusterhead in each cluster. Clusterheads form a DS and no two clusterheads are neighbors. Each clusterhead directly connects to all its members (also called non-clusterheads). The classical clustering algorithm, also called the cluster-based scheme, works as follows.

Cluster formation: (1) A node v is a clusterhead if it has the highest priority in its 1-hop neighborhood including v . (2) A clusterhead and its neighbors form a cluster and these nodes are covered. (3) Repeat (1) and (2) on all uncovered nodes (if any).

Figure 1(c) shows an example of the clustering process. Both s and t are clusterheads (black nodes) since they are local minima (in terms of node ID). u and x belong to cluster s while v and y belong to cluster t . Node w can belong to either s or t . If the node ID of w is changed to m in Figure 1(c), node m is the only clusterhead. When a node has multiple adjacent clusterheads, it belongs to one of them. The cluster formation may need several rounds to complete, depending on the network topology and the priority distribution.

Once the cluster formation process is complete, some non-clusterheads are designated as *gateways* to connect clusterheads. In early schemes [23], every border node (i.e., non-clusterhead that has a neighbor in another cluster) is a gateway, which results in a large CDS. In the tree scheme [4], a clusterhead is first elected as the root. Then the root initiates a flooding to build a rooted tree. In the mesh scheme [3], each clusterhead designates gateways to connect all neighboring clusterheads (i.e., clusterheads within 3 hops). Both the tree and mesh schemes have constant approximation ratios. The tree scheme achieves a better ratio at the expense of slower convergence.

In the core-based approach [13, 28], clusterheads (called core nodes) are permitted to be adjacent, but the core formation can be done in a constant number of rounds without sequential propagation. The original core-based approach is non-deterministic (i.e. time-sensitive depending on when each node participates in the formation process). Here we consider a simplified and deterministic version.

Core formation: A node v becomes a core node if (1) it has the highest priority among its 1-hop neighbors including v (v is selected by itself as a core node), or (2) it has the highest priority based

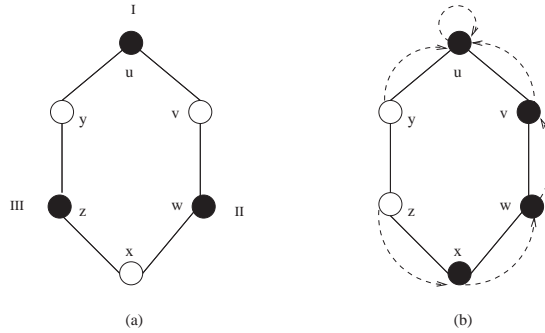


Figure 2: The clustering approach with black nodes as clusterheads in (a) and cores in (b).

on a neighbor's 1-hop neighborhood (v is selected by a neighbor as a core node).

Figure 2 shows the application of both cluster and core formations to the same network. Node degree is used as the priority and node ID is used to break a tie in node degree. In this case, the priority in decreasing order is $u > v > w > x > y > z$. Black nodes are clusterheads/core nodes. In Figure 2(a), each Roman numeral indicates the round number (assume the formation is synchronous) in which the corresponding node is selected as a clusterhead. Each dashed arrow line in Figure 2 indicates the *selector* of each core node. Like clusterheads, core nodes can be connected via gateways to connect neighboring core nodes. To distinguish these two approaches, the former is called a cluster formation, where clusterheads are not adjacent, and the latter is called a core formation.

4 Backbone Formation in Dense Networks

This section proposes a density-reduction approach that can be integrated into any local approach for CDS construction, using MP and Rule k as an example. In the proposed methods, the network density is first reduced using clustering with a short transmission range. Then neighboring clusterheads are connected using a long (and normal) transmission range. In this way clusterheads form a CDS without using gateways. This CDS is further reduced by applying MP and Rule k . Depending on the selection of the short and long transmission ranges, two approaches can be used to construct a backbone. The first approach adopts a 2-level hierarchy: In the lower level, the entire network is covered by the set of clusterheads under the short transmission range. In the upper level, all

clusterheads are covered by the set of *marked clusterheads* under the long transmission range. The second approach constructs a flat backbone, where the entire network is directly covered by the set of marked clusterheads with the long transmission range. For each approach, we show an efficient broadcast scheme as an application.

4.1 2-level clustering approach

We first used different transmission ranges at different stages of the protocol handshake, and then applied the long (and normal) transmission range in broadcasting among clusterheads and the short transmission range in broadcasting within each cluster with an unmarked clusterhead. This approach is similar to the clustering approach that forms a CDS in a dense graph. However, unlike the regular clustering approach where a selection process is needed to select gateway nodes to connect clusterheads, we used a reduced transmission range for clustering. The virtual backbone formation procedure is as follows:

Marking process on clusterheads

1. Each node uses a transmission range of $r/3$ for cluster formation.
 2. Each clusterhead uses a transmission range of r for MP and Rule k .
-

In the above process, the backbone is constructed based on clusterheads using a transmission range of $r/3$. A transmission range of $r/3$ ensures that all neighboring clusterheads (i.e., clusterheads within 3 hops) are directly connected under a transmission range of r .

More formally, we use $G = (V, P(V), r)$ to represent a unit disk graph with node set V , a mapping $P : V \rightarrow R^2$, where R is the real number set, and $r \in R_+$ represents a uniform transmission range from the positive real number set R_+ . P maps each node in V to an (x, y) point in 2-D space. Two nodes are connected if their Euclidean distance is no more than r . G can be simplified to $G(r)$ to represent a unit disk graph with a uniform transmission range of r . It is assumed that $G(r/k)$ is still a connected graph for a small k such as $k = 3$ or 4. This assumption is reasonable under the unit disk graph model when the network is relatively dense and uniformly distributed. These requirements will be relaxed in the next section, where the backbone formation algorithm is extended to non-perfect unit disk graphs with a non-uniform node distribution.

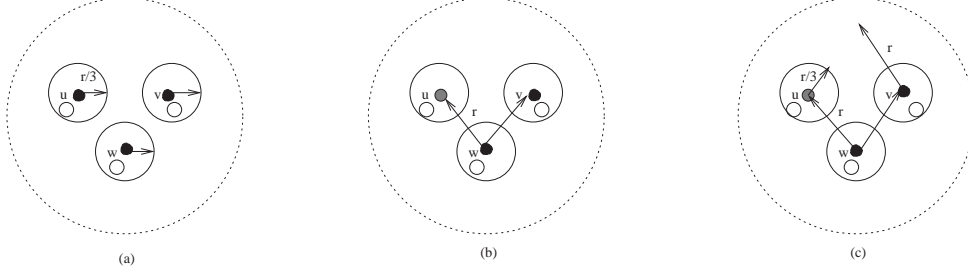


Figure 3: (a) Cluster formation with a transmission range of $r/3$. (b) Marking process with a transmission range of r . (c) Clusterheads forward the broadcast message with different transmission ranges. Marked clusterheads are black, unmarked clusterheads are gray, and non-clusterheads are white.

Lemma 1 *Under the unit disk graph model, a DS of $G(r_1)$ is a CDS of $G(r_2)$, if $G(r_1)$ is connected and $r_2 \geq 3r_1$.*

Proof: Let V' be a DS of $G(r_1)$. An alternative definition of a CDS is that any node pair in the network is connected via nodes in the CDS (i.e., the backbone nodes). For any two nodes u and v , we can construct a path $(u, w_1, w_2, \dots, w_l, v)$ in $G(r_2)$, such that $w_i \in V'$ for $1 \leq i \leq l$. Since $G(r_1)$ is connected, a path $(u = x_1, x_2, \dots, x_l = v)$ exists in $G(r_1)$. For each x_i ($1 \leq i \leq l$), there is a corresponding $w_i \in V'$ that is either x_i itself or a neighbor of x_i . The distance between x_i and w_i is $d(x_i, w_i) \leq r_1$. The distance between w_i and w_{i+1} is $d(w_i, w_{i+1}) \leq d(w_i, x_i) + d(x_i, x_{i+1}) + d(x_{i+1}, w_{i+1}) \leq 3r_1 \leq r_2$. Therefore, $(u, w_1, w_2, \dots, w_l, v)$ is a valid path in $G(r_2)$. \square

The ratio $r_2 = 3r_1$ (or $r_1 = r_2/3$) is tight. A CDS cannot be guaranteed if $r_1 > r_2/3$. On the other hand, using a shorter r_1 will produce a larger clusterhead set, which is undesirable. Because the set of clusterheads is a DS, the following theorem can be proved based on Lemma 1.

Theorem 1 *The clusterhead set V' , derived from $G(r/3)$ via clustering, is a CDS of $G(r)$.*

Let $G'(r)$ be the subgraph of $G(r)$ derived from V' . Since MP and Rule k preserve a CDS, we have:

Corollary 1: *V'' derived from the MP and Rule k is a CDS of $G'(r)$.*

Figure 3(b) illustrates the stage of applying MP and Rule k on clusterheads using a transmission range of r . As a result of the above process, the marked clusterheads form a CDS among clusterheads. The broadcast process is as follows:

Broadcast process

1. If the source is a non-clusterhead, it transmits the message with a transmission range of $r/3$ to the *source clusterhead*.
 2. The source clusterhead transmits the message with a transmission range of r .
 3. At each intermediate node, if the node is a marked clusterhead, it forwards the message with a transmission range of r and if it is an unmarked clusterhead, it forwards the message with a transmission range of $r/3$; otherwise, it does nothing.
-

Theorem 2 *The broadcast process ensures full coverage.*

Proof: Based on the broadcast process, if the source is not a clusterhead, it will forward the message to its clusterhead. Once the message is received by one clusterhead, it will be forwarded by marked clusterheads in V'' to all clusterheads in V' (Corollary 1). Each clusterhead will forward once, using a transmission range of r if it is marked, or a transmission range of $r/3$ if it is unmarked. In either case, each clusterhead will cover all members (non-clusterheads) that are within $r/3$. \square

When the notion of clusterhead coverage is extended to cover clusterheads and all their members, each unmarked clusterhead is still required to forward the message with a transmission range of $r/3$ to ensure coverage within its cluster, because when MP and Rule k are used, the coverage is only extended to all clusterheads, not to all their members which are within $r/3$. Figure 3(c) shows the broadcast process in the 2-level clustering approach.

4.2 1-level flat approach

In the 2-level clustering approach, the broadcast process involves both inter-cluster and intra-cluster broadcast using different transmission ranges. In the 1-level flat approach, the notion of clustering is removed by using a uniform transmission range. Still, different transmission ranges

are used at different stages of the protocol handshake. The modified cluster formation procedure is as follows:

Marking process on clusterheads

1. Each node uses a transmission range of $r/4$ for cluster formation.
 2. Each clusterhead uses a transmission range of $3r/4$ for MP and Rule k .
-

Theorem 1a: *The clusterhead set V' , derived from $G(r/4)$ via clustering, is a CDS of $G(3r/4)$.*

Theorem 1a can be proved in the same way as Theorem 1. Let $G'(3r/4)$ be the subgraph of $G(3r/4)$ derived from V' , we also have

Corollary 1a: *V'' derived from MP and Rule k is a CDS of $G'(3r/4)$.*

Compared with the 2-level clustering approach, shorter transmission ranges are used in the 1-level flat approach for cluster formation and the marking process. As a result, marked clusterheads form a CDS among all nodes in the network. The selection of these transmission ranges is tight. Global domination cannot be guaranteed using larger transmission ranges. The broadcast process is as follows:

Broadcast process

1. The source node and all marked clusterheads forward the broadcast packet using a transmission range of r .
-

Theorem 2a: *The broadcast process ensures full coverage.*

Proof: Based on the broadcast process, each marked clusterhead in V'' forwards the broadcast message. From Corollary 1a, each clusterhead u in V' receives the message from at least one neighboring marked cluster v in $G(3r/4)$. Since the distance between u and v is at most $3r/4$ and the distance between u and all its cluster members in $G(r/4)$ is at most $r/4$, the distance from v to each member of u is at most r . That is, all non-clusterheads also receive the broadcast message. \square

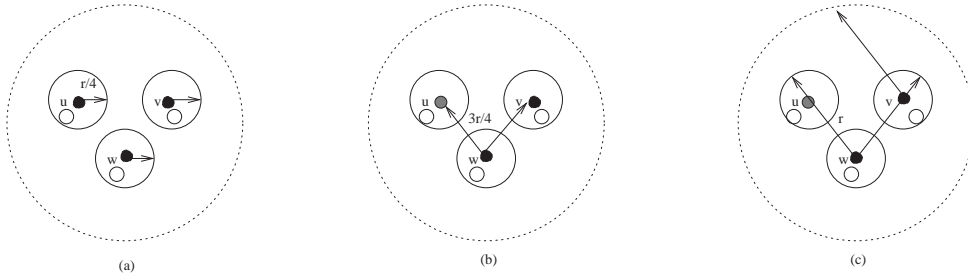


Figure 4: (a) Cluster formation with a transmission range of $r/4$. (b) Reducing the CDS containing clusterheads with a transmission range of $3r/4$. (c) Only marked clusterheads forward the message with a transmission range of r .

Figure 4 shows the 1-level flat approach. Figure 5 illustrates sample backbones constructed via MP and Rule k , the core-based approach, two cluster-based approaches, and two proposed approaches. The sample network is in a 100×100 area with 1000 nodes and a normal transmission range of $r = 24$. MP and Rule k (a) have 72 marked nodes. The core-based approach (b) has 71 core nodes and 60 gateways. The two cluster-based approaches use gateways to connect clusterheads. The size of the CDS is 33 in the tree scheme (c) and 48 in the mesh scheme (d). The 2-level approach (e) selects 98 clusterheads in the first stage, but only 20 marked clusterheads in the second stage. The 1-level flat approach (f) has 156 clusterheads and 43 marked clusterheads.

4.3 Performance analysis

The quality of a backbone is measured by the approximation ratio, i.e., the maximal ratio of the size of the backbone to the size of the minimal CDS. This subsection shows that both approaches have $O(1)$ approximation ratio, and $O(\Delta)$ computation complexity and $O(1)$ message complexity at each node. We also analyze the time steps (or *rounds* of control message exchange) used in the CDS formation. Although the proposed approaches need $O(n)$ rounds in the worst case, we show that they complete in $O(\log n')$ rounds in most cases, where n' is the number of clusterheads and is usually proportional to the area of the 2-D space occupied by a MANET, and inversely proportional to the transmission range.

Both proposed approaches consist of two stages: (1) cluster formation and (2) pruning via MP and Rule k . The $O(1)$ approximation ratio is guaranteed by stage 1 and preserved in stage 2. That

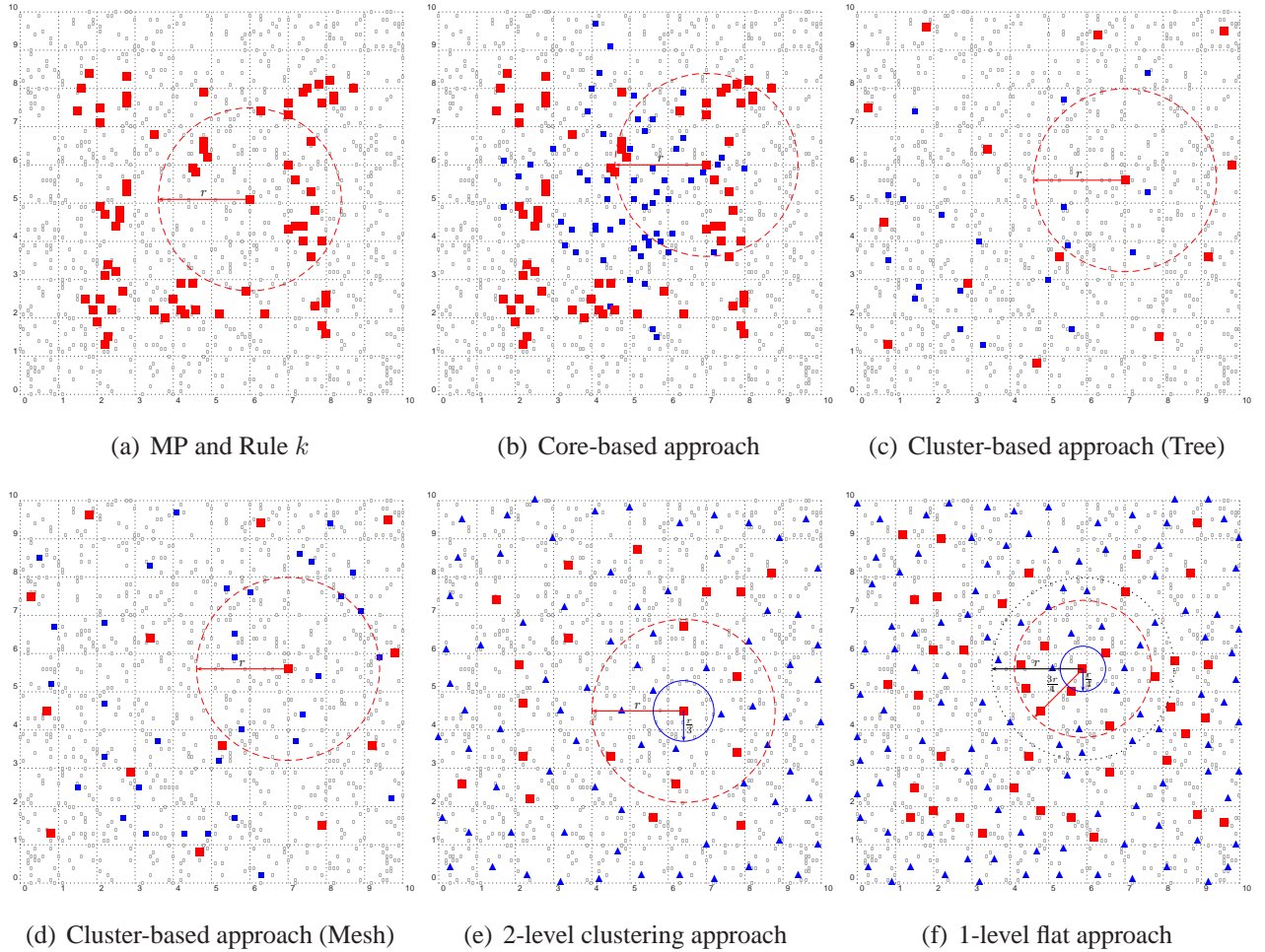


Figure 5: Sample backbones constructed by six CDS algorithms. Nodes in the CDS are marked as squares. Small squares represent gateways in core-based and cluster-based approaches. Small triangles represent unmarked clusterheads in proposed approaches.

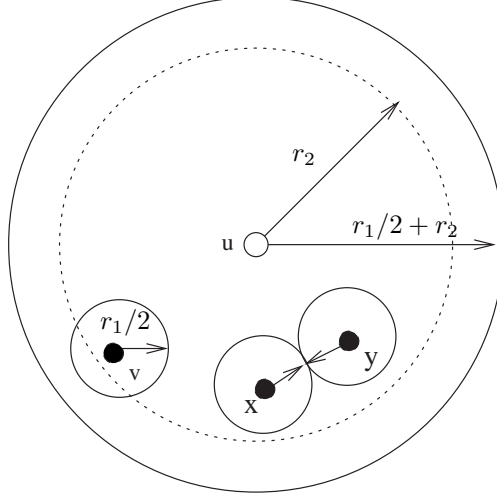


Figure 6: Maximal number of neighboring clusterheads.

is, an upper bound exists on the number of clusterheads in a finite area. Assume transmission range r_1 is used in stage 1 and r_2 in stage 2. We call node v a *neighboring clusterhead* of node u , if v is a clusterhead in stage 1 and within range r_2 of u . The following lemma shows that the number of neighboring clusterheads is bounded by a constant. A similar lemma has been proved in [2]. We include our proof for completeness.

Lemma 2 *Each node has at most $(\frac{r_1 + 2r_2}{r_1})^2$ neighboring clusterheads.*

Proof: For each neighboring clusterhead v of a given node u , draw a circle centered at v with radius $r_1/2$, as shown in Figure 6. Because two clusterheads cannot be neighbors, the distance between any two clusterheads, say x and y , is larger than r_1 . Therefore, those circles with radius $r_1/2$ are non-overlapping. Since the centers of these circles are within range r_2 of u , all these circles are within a large circle centered at u with radius $r_1/2 + r_2$. The total number of neighboring clusterheads of u is no more than the total number of non-overlapping $r_1/2$ circles in the large circle, which is less than $\frac{\pi(r_1/2 + r_2)^2}{\pi(r_1/2)^2} = (\frac{r_1 + 2r_2}{r_1})^2$. \square

Theorem 3 *Both the 2-level clustering and 1-level flat approaches have an $O(1)$ approximation ratio.*

Proof: Suppose V_{opt} is a minimal CDS constructed in an optimal approach. The backbone formed by the 2-level clustering approach consists of both marked and unmarked clusterheads. Note that

each clusterhead is covered by at least one node in V_{opt} . That is, each clusterhead v elected with $r_1 = r/3$ must have a neighbor $u \in V_{opt}$ within distance $r_2 = r$. Based on Lemma 2, each node in V_{opt} can cover at most 49 clusterheads. Therefore, the number of clusterheads is at most 49 times $|V_{opt}|$. In the 1-level flat approach, the backbone uses marked clusterheads only. By applying Lemma 2 with $r_1 = r_2/4$, the number of clusterheads is less than $81|V_{opt}|$, as is the number of marked clusterheads. \square

However, the importance of the approximation ratio, which gives a bound on the worst case performance of a CDS algorithm, should not be overstated. A more important metric, the average performance, should be obtained via probabilistic analysis or simulation study.

Theorem 4 *Both the 2-level clustering and 1-level flat approaches have $O(\Delta)$ computation complexity and $O(1)$ message complexity at each node, where Δ is the maximal node degree under the transmission range used in the cluster formation stage.*

Proof: In the cluster formation stage, each node sends two $O(1)$ messages, the first containing its ID and the second advertising its decision on becoming a clusterhead or non-clusterhead. Each node receives $O(\Delta)$ messages from its neighbors and takes $O(1)$ time in processing each message. Therefore, stage 1 has $O(\Delta)$ computation complexity and $O(1)$ message complexity.

For the pruning stage, it was proved in [10] that both MP and Rule k have $O(\Delta^2)$ computation complexity and $O(\Delta)$ message complexity at each node. As shown in Lemma 2, stage 2 is applied on a sparse graph where $\Delta = O(1)$. Therefore, stage 2 has $O(1)$ time complexity and $O(1)$ message complexity. Overall, both proposed approaches have $O(\Delta)$ computation complexity and $O(1)$ message complexity at each node. \square

We assume a constant length for node ID in Theorem 4. When n is extremely large, it takes $O(\log n)$ bits to represent a unique node ID and $O(\log n)$ time to process each message. In this case, the proposed approaches have $O(\Delta \log n)$ computation complexity and $O(\log n)$ message complexity at each node.

Another measure of the time is the number of rounds of message exchanges. In a MANET with dynamic topology changes, a CDS is formed and maintained via periodic exchange of control messages among neighbors. Due to the interdependence among control messages from different nodes, a CDS formation process usually requires several rounds. For example, MP combined with Rule k completes in two rounds. In the first round, each node advertises its ID. In the second

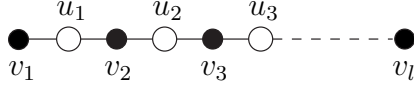


Figure 7: A descending sequence in 1-D space.

round, each node advertises its 1-hop neighbor set built in the last round. Then the status of each node can be determined based on its neighbors' neighbor sets.

Unfortunately, cluster formation may not be complete in constant rounds. Assume clusterheads are elected with minimal node ID. In the best case, stage 1 completes in 3 rounds: After every node advertises its ID, all clusterheads are elected in the second round, and all non-clusterheads announce their status in the third round. In the worst case, stage 1 may take $O(n)$ rounds. As shown in Figure 7, when all nodes form a sequence with decreasing node ID's (i.e., $v_1 > u_1 > v_2 > u_2 \dots > v_l$), the cluster formation process requires $n + 1$ rounds to complete. Node v_1 cannot become a clusterhead until u_1 becomes a non-clusterhead, while before u_1 becomes a non-clusterhead, it must wait for v_2 to become a clusterhead, and so on. Fortunately, the following theorem shows that the situation is much better in the average case.

Theorem 5 *Let K be the number of rounds used in a cluster formation process and n' the number of clusterheads elected, The expectation of K , $E[K] = O(\log n')$.*

The proof of Theorem 5 is in the supplemental material provided with this paper. It shows that in average cases, stage 1 completes in $O(\log n')$ rounds. Since stage 2 requires only two rounds, both proposed approaches complete in $O(\log n')$ rounds in most situations. The number of clusterheads n' in a given 2-D space with area S is bounded by S/r_1^2 , where r_1 is the transmission range used in the first stage. Therefore, both proposed approaches complete in $O(\log \frac{S}{r_1^2})$ rounds on average.

5 Extensions

5.1 A general framework

In this section, we use a more realistic model of MANETs called *quasi-unit disk graph* [16], where the transmission propagation pattern is not a perfect circle. Given a transmission range r , the

corresponding quasi-unit disk graph is still denoted as $G(r)$ under the following definition. A link (u, v) definitely exists in $G(r)$ if its length $d(u, v) \leq r/c$, where $c \geq 1$ is a constant, may or may not exist if $r/c < d(u, v) \leq r$, and must not exist if $d(u, v) > r$. A unit disk graph is a special quasi-unit disk graph with $c = 1$.

Lemma 3 *Under the quasi-unit disk graph model, a DS of $G(r_1)$ is a CDS of $G(r_2)$, if $G(r_1)$ is connected and $r_2 \geq 3cr_1$.*

The proof of the above lemma is similar to that of Lemma 1. It is sufficient to show that the distance between two neighbors in $G(r_1)$ is at most r_1 , and two nodes are connected in $G(r_2)$ when their distance is no longer than r_2/c . Both 2-level clustering and 1-level flat approaches can be generalized into the following 2-stage process:

2-stage backbone formation

1. Each node applies a selected DS algorithm using a transmission range of r_1 to form a DS, V' , of $G(r_1)$.
 2. Each node in V' applies a selected CDS algorithm using a transmission range of $r_2 = 3cr_1$ to form a CDS, V'' , of $G(r_2)$.
-

Here $G'(r_2)$ is the subgraph of $G(r_2)$ induced by V' . In the general framework, DS algorithms other than cluster formation and CDS algorithms other than the marking process (MP) can be used in each stage. For example, the core formation algorithms [13, 28] can be used stage 1. Similarly, multipoint relay (MPR) [26] can be used in stage 2.

The backbone formed by the above process can be used in two broadcast processes. In the 2-level broadcast process, the source node and all nodes in V'' transmit the message with a transmission range of r_2 , and other nodes in V' transmit with range r_1 . In the 1-level broadcast process, the source node and all nodes in V'' transmit the message with a transmission range of $c(r_2 + r_1)$. The correctness of both schemes is guaranteed by Theorem 6.

Theorem 6 *If $G(r_1)$ is connected, V'' is a CDS of both $G'(r_2)$ and $G(c(r_2 + r_1))$.*

Proof: V' produced in stage 1 is a DS of $G(r_1)$. From Lemma 3, V' is a CDS of $G(r_2)$ and $G'(r_2)$ is connected. Therefore, V'' produced in stage 2 is a CDS of $G'(r_2)$. To prove the second part of the theorem, it is sufficient to show that V'' is a DS of $G(c(r_2 + r_1))$. Note that every node v in the network has a node $v' \in V'$ with the distance $d(v, v') \leq r_1$, which has a node $v'' \in V''$ within the distance $d(v', v'') \leq r_2$. Therefore, $d(v, v'') \leq r_2 + r_1$ of v'' , and v is dominated by V'' in $G(c(r_2 + r_1))$. \square

When cluster formation is used to construct a DS in stage 1, Lemma 2 still holds in a quasi-unit disk graph after a minor modification. Since two clusterheads cannot be neighbors, the minimum distance between two nodes in the DS is r_1/c . Therefore, there are at most $(\frac{r_1 + 2cr_2}{r_1})^2$ clusterheads in a disk with radius r_2 . Consequently, the 2-stage backbone formation process has the same asymptomatic approximation ratio and time and message complexity as the original scheme.

5.2 Recursive density reduction

In very dense networks, a large average node degree causes high contention and computation cost in stage 1. We further generalize the 2-stage process into the following k -stage process, which reduces node degree in stage 1 using a smaller r_1 .

k -stage backbone formation

1. Each node uses a transmission range of r_1 to form a DS, V_1 , of $G(r_1)$.
 2. Each node uses a transmission range of $r_2 = 3cr_1$ to form a DS, V_2 , of $G_1(r_2)$.
 - ...
 - k . Each node in V_{k-1} uses a transmission range of $r_k = 3cr_{k-1}$ to form a CDS, V_k , of $G_{k-1}(r_k)$.
-

Here $G_i(r_{i+1})$ ($1 \leq i \leq k - 1$) is the subgraph of $G(r_{i+1})$ induced by V_i . For $k > 2$, the recursive density reduction mechanism incurs lower energy and bandwidth overhead than the 2-stage scheme, because most nodes are eliminated in the early stage of the protocol handshake using a small transmission range. The resultant backbone can be used in both hierarchical routing (as

demonstrated by the k -level broadcast process) and flat routing (as demonstrated by the 1-level broadcast process).

k -level broadcast process

1. The source node and all nodes in V_k transmit the message with a transmission range of r_k .
2. All other nodes in V_{k-1} transmit the message with a transmission range of r_{k-1} .
- ...
- k . All other nodes in V_1 transmit the message with a transmission range of r_1 .

1-level broadcast process

1. The source node and all nodes in V_k transmit the message with a transmission range of $c(r_k + r_{k-1} + \dots + r_1)$.
-

Theorem 7 *If $G(r_1)$ is connected, V_k is a CDS of both $G_{k-1}(r_k)$ and $G(c(r_k + r_{k-1} + \dots + r_1))$.*

Proof: When $G(r_1)$ is connected, V_1 is a DS of $G(r_1)$ and $G_1(r_2)$ is connected (Lemma 3). Similarly, V_2 is a DS of $G_1(r_2)$ and $G_2(r_3)$ is connected, and so on. Finally, V_{k-1} is a DS of $G_{k-2}(r_{k-1})$ and $G_{k-1}(r_k)$ is connected. Therefore, V_k is a CDS of $G_{k-1}(r_k)$. In addition, every node v in the network has a node $v_1 \in V_1$ within the transmission range r_1 , which has a node $v_2 \in V_2$ within the transmission range r_2 , and so on. Finally, v is within the transmission range $r_k + r_{k-1} + \dots + r_1$ of a $v_k \in V_k$, which means that V_k is a CDS of $G(c(r_k + r_{k-1} + \dots + r_1))$. \square

The correctness of the above broadcast schemes is guaranteed by Theorem 7. In the k -level broadcast process, the message transmitted by the source will be forwarded by nodes in V_1, V_2, \dots, V_{k-1} in sequence, and finally reaches a node in V_k . Then it will be forwarded by all nodes in V_k and reaches all nodes in V_{k-1} and so on, until it is forwarded by all nodes in V_1 , which covers the entire network. In the 1-level broadcast process, the message is forwarded by all nodes in V_k , which covers the entire network under the transmission range $c(r_k + r_{k-1} + \dots + r_1)$.

The stage number k depends on the global node density. When the nodes are uniformly distributed in a given deployment area, the minimum transmission range r_1 that achieves global connectivity with high probability can be estimated based on the node number and deployment area

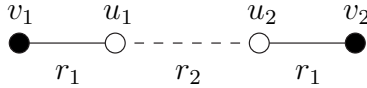


Figure 8: The partition problem.

[33]. We assume each node obtains the knowledge of global density before or right after the deployment. When such knowledge is unavailable, or the nodes are not uniformly distributed, an adaptive density reduction scheme, which will be discussed in the next subsection, can be used to determine the level of each node based on local information.

5.3 Adaptive DS formation

In the previous discussion, we assume the network is connected under a short transmission range r_1 ; otherwise, a *partition problem* exists in the DS formation process. As shown in Figure 8, when the network is connected under a large transmission range (r_2) but disconnected under r_1 , the resultant DS may be disconnected even under the large transmission range. The following connectivity preserving enhancement is applied in stages $1, 2, \dots, k - 1$ of the k -stage scheme to avoid the partition problem.

Adaptive DS formation

1. Each node v determines its initial range r_v using a localized topology control scheme.
 2. At each stage i ($1 \leq i \leq k - 1$), all nodes v with $r_v > r_i$ are automatically added to the DS.
-

The localized topology control scheme can be any method that determines a minimal transmission range assignment to maintain global connectivity. The *expanding search region* scheme [5, 19] can be integrated into the iterative process as follows without extra message overhead: At each stage i , each node sends at least one message in the DS formation process. By receiving these messages, each node collects angle-of-arrival (AoA) information of nodes within range r_i , and determines whether its minimal transmission range is larger than r_i via cone-based topology control [19]. When AoA information is unavailable, each node can determine its minimal transmission range by counting the number of visible neighbors [5], and preserves global connectivity with high

probability. When the above enhancement is applied to the network in Figure 8, both nodes u_1 and u_2 will be added to V_1 , maintaining connectivity under range r_2 .

Theorem 8 *If $G(r_k)$ is connected, then all V_i ($1 \leq i \leq k$) are connected under transmission range r_k .*

Proof: Let V_0 be the set of all nodes in the network. Obviously V_0 is connected under transmission range r_k . Assume V_{i-1} ($0 \leq i \leq k-1$) is connected under transmission range r_k . For any two nodes u and v in V_i , a path $P : (u = w_1, w_2, \dots, w_l = v)$ exists in $G(r_k)$ such that $w_j \in V_{i-1}$ for all $1 \leq j \leq l$. In addition, for each hop (w_j, w_{j+1}) , either $d(w_j, w_{j+1}) \leq r_i$, or both r_{w_j} and $r_{w_{j+1}}$ are larger than $d(w_j, w_{j+1}) > r_i$. For each w_i , let x_j be w_j if $w_j \in V_i$; otherwise, let x_j be a neighbor of w_j in V_i within distance r_i . Consider the new path $P' : (u = x_1, x_2, \dots, x_l = v)$. For each hop (x_j, x_{j+1}) , if $d(w_j, w_{j+1}) \leq r_i$ then $d(x_j, x_{j+1}) < 3r_i < r_k/c$; otherwise, both w_j and w_{j+1} belong to V_i and link $(x_j, x_{j+1}) = (w_j, w_{j+1})$ exists in $G(r_k)$. In both cases, P' is a valid path in $G(r_k)$. That is, V_i is connected under range r_k . In the final stage ($i = k$), a CDS algorithm, which preserves connectivity, is applied to V_{k-1} . Therefore, V_k is also connected under range r_k . \square

In the adaptive DS formation process, the resultant DS may contain neighboring nodes. That is, Lemma 2 and the constant approximation ratio and message complexity may not hold. However, these properties are guaranteed if the cluster formation process is used in k -stage backbone formation, and each node v has its $r_v \leq r_{k-1}$.

6 Simulation

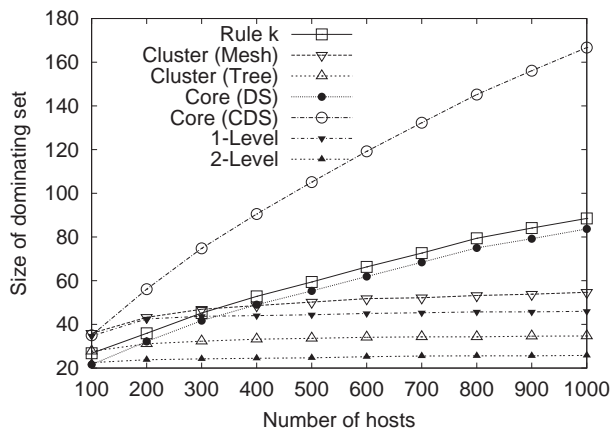
The efficiency and overhead of both proposed approaches are evaluated via simulations. The 2-level clustering approach (2-Level) and the 1-level flat approach (1-Level) are compared with several existing ones, including the combination of MP and Rule k (Rule k), two cluster-based approaches using a mesh (Mesh) and a tree (Tree) to connect clusterheads, and the core-based approach (Core). In the 2-level approach, the resultant CDS is a dominating set of the subnetwork consisting of clusterheads. For Core, two versions are considered: one for the DS consisting of core nodes only (DS) and another for the CDS consisting of both core nodes and non-core nodes in forwarding sets (CDS). We use node ID as priority in cluster formation to reduce the number

of messages (2 messages per node for node ID while 3 for node degree) and energy consumption. Since Rule k can use node degree as priority with 2 messages per node, node degree is used to improve pruning performance. The core formation process also uses node degree as priority, which is an approximation of the effective degree (i.e., number of selectors) used in Core.

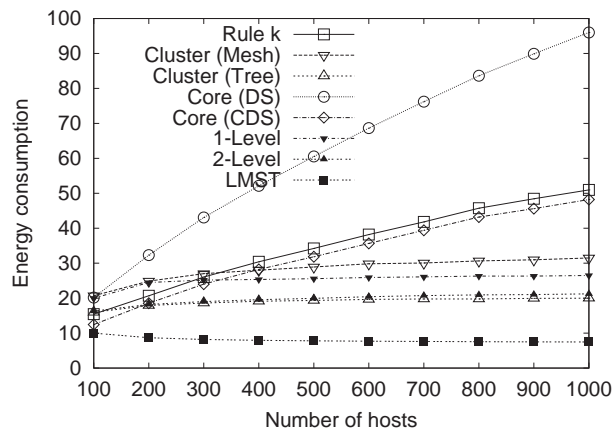
All approaches are simulated on a custom simulator. In order to generate a random network, n nodes are randomly placed in a 100×100 square region to form a unit disk graph using a transmission range of r . For Rule k , Mesh, Tree, and Core, r is set to 24. For the 2-level approach, r is 8 in the first (clustering) stage and 24 in the second (pruning) stage. For the 1-level approach, r is 6 in the first stage and 18 in the second stage. Each simulation is repeated until the 90% confidence interval is within $\pm 1\%$.

Efficiency: We compare the efficiency of different approaches in terms of the size of the resultant CDS, and the energy consumption in the corresponding broadcast process. Figure 9(a) shows the size of the resultant backbone in different approaches. In Rule k , the CDS size increases rapidly as the network size (n) grows. The size of the DS in Core is very close to the size of the CDS in Rule k , and the size of the CDS in Core is much larger than in other approaches. In other words, neither Rule k nor Core is very efficient in dense networks. In other approaches, the CDS sizes are barely affected by the network density. For $n \geq 500$, increasing n can cause only a slight difference in the CDS sizes. The CDS sizes in those approaches depend on the number of clusterheads, which has a constant upper bound in a region with a fixed size. Among those approaches, the 1-level approach is about 20% better than the mesh approach, and the 2-level approach is about 30% better than the tree approach. Although the 2-level approach produces a smaller CDS than the 1-level approach, it also requires a more complex routing scheme.

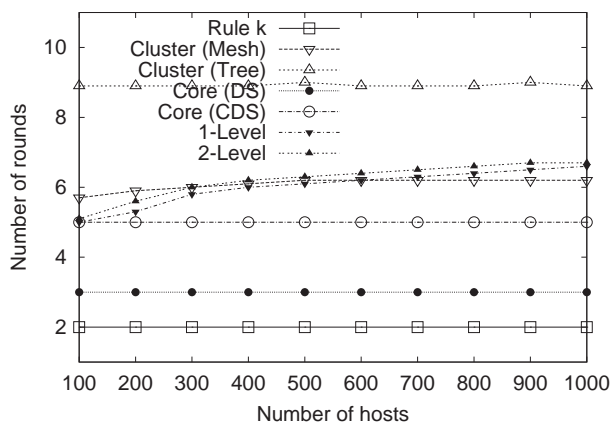
Figure 9(b) shows the broadcast cost of different approaches in terms of the total transmission power. In the 2-level approach, all marked nodes transmit the message with the normal transmission range r , and all unmarked nodes transmit with a transmission range of $r/3$. In other approaches, all backbone nodes transmit with the normal transmission range. A commonly used energy model [12] can be stated as $e = \alpha r^k + \beta$, where e is the energy consumption, k is usually between 2 and 4, and α, β are device specific constants. Here we use $k = 2$, $\alpha = 0.001$, and $\beta = 0$. The result is quite similar to the case of CDS size. The only difference is that, after considering the energy consumption of unmarked clusterheads, the broadcast cost of the 2-level approach is slightly higher than that of the tree approach, but still significantly lower than the other



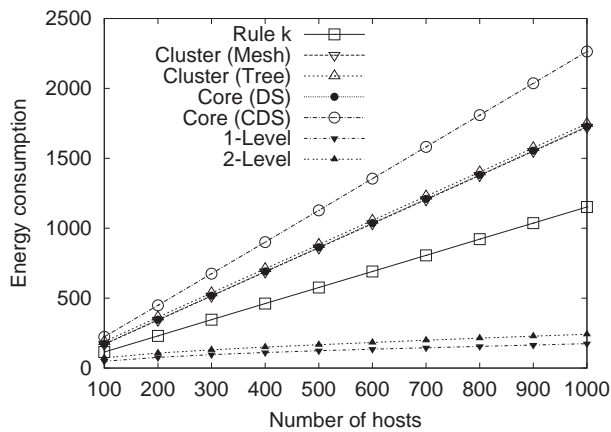
(a) Size of CDS



(b) Broadcast cost



(c) Number of rounds



(d) Energy consumption on the senders' side.

Figure 9: Simulation results.

approaches. For comparison, a topology control algorithm based on local minimal spanning tree (LMST) [20] is also simulated, assuming all nodes forward the broadcast packet with a small transmission power. The broadcast cost of LMST is about 50% of the best CDS algorithm. Note that the relative performance of a topology control scheme and a CDS-based scheme depends on the underlying energy model. The topology control approach is better with large α and k . The CDS approach is superior with a large β .

Overhead: Two types of overhead are considered in our comparison: time, and energy. We measure the time cost in terms of the number of rounds of message exchange. Rule k completes in 2 rounds. In Core, core formation requires 3 rounds, and the designation of forwarding sets needs 2 extra rounds. In other approaches, more rounds are required to obtain a stable cluster structure. After clusterhead formation, both 1-level and 2-level approaches require two extra rounds to apply MP and Rule k . The mesh approach also requires two extra rounds: one for gathering neighboring cluster information, and another for gateway designation. The tree approach has two extra phases: root election and tree construction via flooding. Here we assume that the root is pre-selected and consider only the flooding cost. As shown in Figure 9(c), Rule k and Core have the lowest cost, and the tree approach has the highest cost. The 1-level, 2-level, and mesh approaches have similar costs. That is, both proposed approaches achieve higher efficiency than the mesh approach without extra time cost.

Considering the different transmission powers for different transmission ranges (r), the energy consumption of the two proposed approaches is much lower than the other approaches. In the clustering stage of the two proposed approaches, packets are sent to a smaller transmission range, which is only $1/3$ or $1/4$ of the normal transmission range. Figure 9(d) shows the energy consumption during the backbone formation process. The energy consumption of both proposed approaches is a fraction of the other approaches.

Simulation results can be summarized as follows: (1) Both proposed approaches produce a smaller CDS than Rule k , Core and the mesh approach. (2) Both proposed approaches have a converging speed similar to that of the mesh approach, which is significantly faster than the tree approach. (3) Both proposed approaches have significantly lower energy consumption than Rule k , Core, mesh, and tree approaches.

7 Conclusions

We have proposed a novel approach to address the communication and computation complexity issue in many local CDS construction algorithms. This approach is based on a special method of merging the clustering approach with the use of different transmission ranges. Wu and Li's marking process has been extended as an illustration of the proposed approach. Specifically, the clustering algorithm is applied using a short transmission range to reduce the density of a MANET. Clusterheads form a connected dominating set (CDS) using a long transmission range, which can be used as a backbone of the MANET. Wu and Li's marking process is then applied to the CDS to reduce the number of backbone nodes.

Two routing schemes have been proposed based on the backbone formation approach. In the 2-level hierarchical approach, messages are transmitted using a long transmission range by a small set of selected clusterheads that form the upper level backbone, and a short transmission range by other clusterheads that form the lower level backbone. In the 1-level flat approach, messages are transmitted by only selected clusterheads using a long transmission range. The 2-level approach is more energy efficient, as fewer nodes use the long transmission range. The 1-level approach has a simpler routing process and uses fewer backbone nodes. Both analytic and simulation studies confirm the effectiveness of the proposed approaches, especially in dense networks.

We have further extended the proposed approach to a general framework that uses other existing clustering and CDS formation algorithms, including the core-based approach [13, 28] and MPR [1, 26], for tradeoffs between the number of backbone nodes and various formations' overhead. In very dense networks, we have proposed multi-stage density reduction, which uses different transmission ranges in different stages of the backbone formation process to control the communication and computation cost of each stage. Our future work will focus on other applications of the virtual backbone, including topology management in MANETs and point and area coverage in sensor networks.

References

- [1] C. Adjih, P. Jacquet, and L. Viennot. Computing connected dominated sets with multipoint relays. Technical Report 4597, INRIA-Rapport de recherche, Oct. 2002.

- [2] K. Alzoubi, X. Y. Li, Y. Wang, P. J. Wan, and O. Frieder. Geometric spanners for wireless ad hoc networks. *IEEE Transactions on Parallel and Distributed Systems*, 14(5):408–421, 2003.
- [3] K. M. Alzoubi, P. J. Wan, and O. Frieder. Message-optimal connected dominating sets in mobile ad hoc networks. In *Proc. of MobiHoc*, pages 157–164, June 2002.
- [4] K. M. Alzoubi, P. J. Wan, and O. Frieder. New distributed algorithm for connected dominating set in wireless ad hoc networks. In *Proc. of HICSS-35*, page 297, Jan. 2002.
- [5] D. Blough, M. Leoncini, G. Resta, and P. Santi. The K-Neigh protocol for symmetric topology control in ad hoc networks. In *Proc. of MobiHoc*, pages 141–152, June 2003.
- [6] M. Burkhard, P. Rickenbach, R. Wattenhofer, and A. Zollinger. Does topology control reduce interference. In *Proc. of MobiHoc*, 2004.
- [7] J. Cartigny, D. Simplot, and I. Stojmenovic. Localized minimum-energy broadcasting in ad-hoc networks. In *Proc. of Infocom*, pages 2210–2217, 2003.
- [8] B. Chen, K. Jamieson, H. Balakrishnan, and R. Morris. SPAN: an energy-efficient coordination algorithm for topology maintenance in ad hoc wireless networks. *ACM Wireless Networks Journal*, 8(5):481–494, 2002.
- [9] A. Clementi, P. Crescenzi, P. Penna, G. Rossi, and P. Vocca. On the complexity of computing minimum energy consumption broadcast subgraphs. In *Proc. of STACS*, 2001.
- [10] F. Dai and J. Wu. Distributed dominant pruning in ad hoc wireless networks. In *Proc. of ICC*, volume 1, pages 353–357, May 2003.
- [11] D. Dubhashi, A. Mei, A. Panconesi, J. Radhakrishnan, and A. Srinivasan. Fast distributed algorithms for (weakly) connected dominating sets and linear-size skeletons. In *Proceedings of ACM-SIAM SODA*, pages 717–724, Jan. 2003.
- [12] L. M. Feeney. An energy-consumption model for performance analysis of routing protocols for mobile ad hoc networks. *Mobile Networks and Applications*, 6(3):239–249, June 2001.
- [13] J. Gao, L. J. Guibas, J. Hershberger, L. Zhang, and A. Zhu. Discrete mobile centers. In *Symposium on Computational Geometry*, pages 188–196, 2001.

- [14] S. Guha and S. Khuller. Approximation algorithms for connected dominating sets. *Algorithmica*, 20(4):374–387, Apr. 1998.
- [15] L. Jia, R. Rajaraman, and T. Suel. An efficient distributed algorithm for constructing small dominating sets. In *Proceedings of ACM PODC*, pages 33–42, Aug. 2001.
- [16] F. Kuhn, T. Moscibroda, and R. Wattenhofer. Initializing newly deployed ad hoc and sensor networks. In *Proc. of MobiCom*, pages 260–274, Sep./Oct. 2004.
- [17] F. Kuhn, T. Moscibroda, and R. Wattenhofer. What cannot be computed locally! In *Proc. of PODC*, July 2004.
- [18] F. Kuhn and R. Wattenhofer. Constant-time distributed dominating set approximation. In *Proc. of PODC*, 2003.
- [19] L. Li, J. Y. Halpern, V. Bahl, Y. M. Wang, and R. Wattenhofer. Analysis of a cone-based distributed topology control algorithm for wireless multi-hop networks. In *Proc. of PODC*, pages 264–273, Aug. 2001.
- [20] N. Li, J. C. Hou, and L. Sha. Design and analysis of an mst-based topology control algorithm. In *Proc. of Infocom*, volume 3, pages 1702–1712, Mar./Apr. 2003.
- [21] X. Y. Li, Y. Wang, P. J. Wan, W. Z. Song, and O. Frieder. Localized low-weight graph and its application in wireless ad hoc networks. In *Proc. of Infocom*, 2004.
- [22] H. Lim and C. Kim. Flooding in wireless ad hoc networks. *Computer Communications Journal*, 24(3-4):353–363, 2001.
- [23] C. R. Lin and M. Gerla. Adaptive clustering for mobile wireless networks. *IEEE Journal on Selected Areas in Communications*, 15(7):1265–1275, 1996.
- [24] W. Lou and J. Wu. On reducing broadcast redundancy in ad hoc wireless networks. *IEEE Transactions on Mobile Computing*, 1(2):111–122, Apr.-June 2002.
- [25] W. Peng and X. Lu. On the reduction of broadcast redundancy in mobile ad hoc networks. In *Proc. of MobiHoc*, pages 129–130, June 2000.
- [26] A. Qayyum, L. Viennot, and A. Laouiti. Multipoint relaying for flooding broadcast message in mobile wireless networks. In *Proc. of HICSS-35*, page 298, Jan. 2002.

- [27] V. Rodoplu and T. H. Meng. Minimum energy mobile wireless networks. *IEEE Journal of Selected Areas in Communications*, 17(8):1333–1344, Aug. 1999.
- [28] P. Sinha, R. Sivakumar, and V. Bharghavan. Enhancing ad hoc routing with dynamic virtual infrastructures. In *Proc. of Infocom*, pages 1763–1772, Apr. 2001.
- [29] I. Stojmenovic, S. Seddigh, and J. Zunic. Dominating sets and neighbor elimination based broadcasting algorithms in wireless networks. *IEEE Transactions on Parallel and Distributed Systems*, 13(1):14–25, Jan. 2002.
- [30] J. Sucec and I. Marsic. An efficient distributed network-wide broadcast algorithm for mobile ad hoc networks. CAIP Technical Report 248, Rutgers University, Sep. 2000.
- [31] Y. C. Tseng, S. Y. Ni, Y. S. Chen, and J. P. Sheu. The broadcast storm problem in a mobile ad hoc network. *Wireless Networks*, 8(2-3):153–167, Mar.-May 2002.
- [32] P. J. Wan, G. Calinescu, X. Y. Li, and O. Frieder. Minimum-energy broadcast routing in static ad hoc wireless networks. *ACM Wireless Networks*, 2002.
- [33] P. J. Wan and C. W. Yi. Asymptotic critical transmission radius and critical neighbor number for k -connectivity in wireless ad hoc networks. In *Proc. of MobiHoc*, May 2004.
- [34] W. Z. Song, Y. Wang, X. Y. Li, and O. Frieder. Localized algorithms for energy efficient topology control in wireless ad hoc networks. In *Proc. of MobiHoc*, 2004.
- [35] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides. On constructing minimum spanning trees in k -dimensional spaces and related problems. In *Proc. of Infocom*, pages 585–594, 2000.
- [36] J. Wu and F. Dai. A generic distributed broadcast scheme in ad hoc wireless networks. In *Proc. of ICDCS*, pages 460–468, May 2003.
- [37] J. Wu and H. Li. On calculating connected dominating set for efficient routing in ad hoc wireless networks. In *Proc. of DialM*, pages 7–14, 1999.
- [38] J. Wu and W. Lou. Forward-node-set-based broadcast in clustered mobile ad hoc networks. *Wireless Communications and Mobile Computing, special issue on Algorithmic, Geometric, Graph, Combinatorial, and Vector*, 3(2):155–173, 2003.