Journal of Interconnection Networks

ON LOCALITY OF DOMINATING SET IN AD HOC NETWORKS WITH SWITCH-ON/OFF OPERATIONS*

JIE WU and FEI DAI
Department of Computer Science and Engineering
Florida Atlantic University
Boca Raton, FL 33431, USA
{*jie, fdai*}*@cse.fau.edu*

Received (received date)
Revised (revised date)

*This work was supported in part by NSF grant CCR 9900646 and grant ANI 0073736.

Efficient routing among a set of mobile hosts is one of the most important functions in ad hoc wireless networks. Routing based on a connected dominating set is a promising approach, where the search space for a route is reduced to the hosts in the set. A set is dominating if all the hosts in the system are either in the set or neighbors of hosts in the set. In this paper, we first review a distributed formation of a connected dominating set called *marking process* and dominating-set-based routing. Then we propose several ways to reduce the size of the dominating set and study the locality of dominating set in ad hoc wireless networks with switch-on/off operations. Results show that the dominating set derived from the marking process exhibits good locality properties; i.e., the change of a host status, gateway (dominating) or non-gateway (dominated), affects only the status of hosts in a restricted vicinity. In addition, locality of host status update is also verified through simulation.

Keywords: Ad hoc wireless networks, dominating sets, mobility management, routing, simulation.

1. Introduction

Recent advances in technology have provided portable computers with wireless interfaces that allow networked communication among mobile users. The resulting computing environment, which is often referred to as mobile computing, no longer requires users to maintain a fixed and universally known position in the network and enables almost unrestricted mobility. It is argued that future wireless will be converged to be more ad hoc and reconfigurable [7]. An *ad hoc wireless network* (or simply *ad hoc network*) [10] is a special type of wireless mobile networks in which a collection of mobile hosts with wireless network interfaces forms a temporary network, without the aid of any established infrastructure (i.e., base stations) or centralized administration (i.e., mobile switching centers). The applications of ad hoc networks range from civilian use to disaster recovery (search-and-rescue) and military use (battlefield). Commercial and military examples of ad hoc networks include Ricochet [12] and the Army Near-Term Digital Radio (NTDR) [13].

We can use a simple graph $G = (V, E)$ to represent an ad hoc network, where V represents a set of wireless mobile hosts and E represents a set of edges. An edge between a host pair (u, v) indicates that both hosts u and v are within their wireless transmitter ranges; that is, connections of hosts are based on geographic distances of hosts. Thus, the corresponding graph is an undirected graph and it is called a *unit disk graph* [4]. Figure 1 shows a unit disk graph of six hosts, where each cycle with center u corresponds to the transmitter range of host u.

Dominating-set-based routing [22] is a promising routing approach in ad hoc networks. A subset of the vertices of a graph is a dominating set if every vertex not in the subset is adjacent to at least one vertex in the subset. Moreover, this dominating set should be connected for ease of the routing process within the induced graph defined to consist of dominating vertices only. Vertices in a dominating set are also called *gateway* hosts while vertices that are outside a dominating set are called *non-gateway* hosts. In Figure 1, hosts v and w form a connected dominating set of the given unit disk graph.

The main advantage of dominating-set-based routing is that it simplifies the routing process to the one in a smaller subnetwork generated from the connected dominating set. This means that only gateway hosts need to keep routing information in a *proactive approach* and the search space is reduced to the dominating set in a *reactive approach*. In proactive routing, routes to all destinations are computed *a priori* and are maintained in the background via a periodic update process. In reactive routing, a route to a specific destination is computed “on demand”; i.e., only when needed. Note that gateway hosts are used not only to route packets but also to disseminate routing information. Clearly, the efficiency of this approach depends largely on the process of finding and maintaining a connected dominating set and the size of the corresponding subnetwork. Ideally, the connected dominating set should be constructed in a localized manner, where the role of a node can be determined based on the local information like its neighborhood topology. A localized

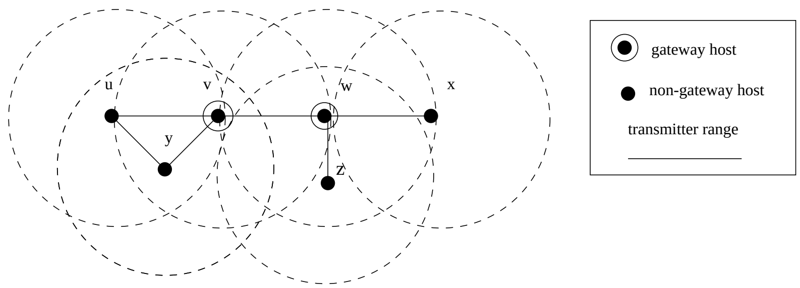

Figure 1: A sample unit disk graph representing an ad hoc network.

protocol guarantees fast convergence speed, which is an essential issue for routing in ad hoc networks.

Unfortunately, finding a minimum connected dominating set is NP-complete for most graphs. Wu and Li [22] proposed a simple and efficient distributed algorithm that can quickly determine a connected dominating set in ad hoc networks. This approach uses a localized algorithm called *marking process* where hosts interact with others in the neighborhood. Each host performs exceedingly simple tasks of maintaining and propagating information *markers*. Specifically, each host is marked true if it has two unconnected neighbors. It is shown that collectively these hosts achieve a desired global objective – a set of marked hosts forms a small connected dominating set. This approach also outperforms several classical approaches, such as the cluster approach [3,9] and MCDS (minimum connected dominating set that forms a backbone) [8,17], in terms of finding a small connected dominating set and does so quickly [21].

In this paper, we focus on maintaining the dominating set in an ad hoc network where switch-on/off operations are major operations that change network topology. Such a network can be either a *sensor network* [6] with limited mobility or a *rooftop network* [14] without mobility, but is deployed very densely in metropolitan areas. The network can be characterized as a large number of hosts operated under a self-organizing way [1]. Each host may be battery constrained or subject to hostile environments so that individual device failure (considered as a switch-off operation) will be a regular event. The limitation of the power of each host poses a unique challenge for power-aware design [20,18]. There has been an increasing focus on low cost and reduced host power consumption in ad hoc networks. Methods include reducing energy consumption by exploiting both sleep state and active power management [15]. Even in standard networks such as IEEE 802.11, requirements are included to sacrifice performance in favor of reduced power consumption [5]. Many of power consumption methods can be abstracted into a switch-off operation on a host which can return to operation later through a switch-on operation.

We study the locality of dominating set in ad hoc networks with switch-on/off operations. The dominating set under consideration is derived from the marking process and it is further reduced through different reduction methods proposed in this paper. The main contributions of the paper include the locality property of the marking process. That is, the change of a host status, gateway (dominating) or non-gateway (dominated), affects only the status of hosts in a restricted vicinity. In addition, locality of host status update is also verified through simulation. We show the different locality properties of gateway/non-gateway derived by different versions of the marking process.

The paper is organized as follows: Section 2 reviews the formation of a connected dominating set using the marking process and basic ideas of dominating-set-based routing. Section 3 discusses different ways to reduce the size of the connected dominating set derived from the marking process. Section 4 focuses on updates of dominating set under switch-on/off operations. Locality of different updates for different versions of the marking process is also discussed. The marking process together with a particular reduction method corresponds to a *version*. Section 5 presents simulation results on locality. The paper concludes in Section 6. Throughout the paper, we use terms host and vertex interchangeably. Similarly, for link and edge and for network and graph.

2. Preliminaries

We first review the decentralized formation of a dominating set through the marking process followed by dominating-set-based routing. Some desirable features for such a process are listed below: (1) The formation process should be distributed and simple. Ideally, it requires only local information and a constant number of iterative rounds of message exchanges among neighboring hosts. (2) The resultant dominating set should be connected and close to minimum. (3) It is desirable that the resultant dominating set includes all intermediate hosts of any shortest path. In this case, an all-pair shortest paths algorithm only needs to be applied to the subnetwork generated from the dominating set.

2.1. *Marking process*

Marking process [22]

1. Initially assign marker F to each v in V.
2. Each v exchanges its neighbor set $N(v)$ with all its neighbors.
3. Each v assigns its marker $m(v)$ to T if there exist two unconnected neighbors.

The marking process is a localized algorithm, where hosts only interact with others in the neighborhood. The marking process marks every vertex in a given connected simple graph $G = (V, E)$. $m(v)$ is a marker for vertex $v \in V$, which is either T (marked) or F (unmarked). We will show later that marked vertices form a connected dominating set. We assume that all vertices are unmarked initially. $N(v) = \{u | (v, u) \in E\}$ represents the neighbor set of vertex v. Each vertex v always maintains its neighbor set $N(v)$.

In Figure 1, $N(u) = \{v, y\}$, $N(v) = \{u, w, y\}$, $N(w) = \{v, x, z\}$, $N(x) = \{w\}$, $N(y) = \{u, v\}$, and $N(z) = \{w\}$. After Step 2 of the marking process, vertex u has $N(v)$ and $N(y)$, v has $N(u)$, $N(w)$, and $N(y)$, w has $N(v)$, $N(x)$, and $N(z)$, x has $N(w)$, y has $N(u)$ and $N(v)$, and z has $N(w)$. Based on Step 3, only vertices v and w are marked T. Clearly, each vertex knows 2-hop neighborhood information after Step 2 of the marking process; i.e., neighbor information of its neighbors'.

2.2. *Properties*

Assume that V' is the set of vertices that are marked T in V; i.e., $V' = \{v | v \in V, m(v) = T\}$. The induced graph G' is the subgraph of G induced by V'; i.e., $G' = G[V']$. The following three results [22] show that G' is a connected dominating set of G.

Property 1 *Given a graph $G = (V, E)$ that is connected but not completely connected, the vertex subset V', derived from the marking process, forms a dominating set of G.*

When G is completely connected, all vertices are marked F. This is desirable, because if all vertices are directly connected, there is no need for gateway hosts.

Property 2 *The induced graph $G' = G[V']$ is a connected graph.*

The next property shows that, except for source and destination vertices, all intermediate vertices in a shortest path are contained in the dominating set derived from the marking process.

Property 3 *The shortest path between any two vertices does not include any non-gateway vertex as an intermediate host.*

Since the problem of determining a minimum connected dominating set of a given connected graph is NP-complete, the connected dominating set derived from the marking process is normally non-minimum. In some cases, the resultant dominating set is *trivial*; i.e., $V' = V$ or $V' = \{\}$. For example, any vertex-symmetric graph will generate a trivial dominating set using the proposed marking process. However, the marking process is efficient for ad hoc networks where the corresponding unit disk graph tends to form a set of localized clusters (or cliques).

2.3. *Dominating-set-based routing*

Assume that a connected dominated set has been determined for a given ad hoc network. *Dominating-set-based routing* usually consists three steps (as shown in Figure 2):

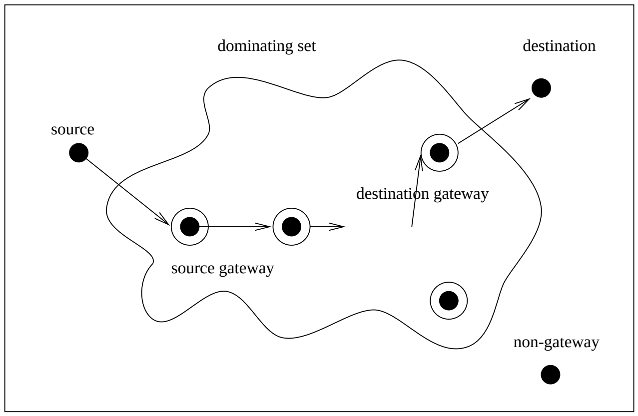

Figure 2: Dominating-set-based routing.

1. If the source is not a gateway host, it forwards the packets to a *source gateway*, which is one of the adjacent gateway hosts.

2. This source gateway acts as a new source to route the packets in the *induced graph* generated from the connected dominating set.

3. Eventually, the packets reach a *destination gateway*, which is either the destination host itself or a gateway that connects the destination host. In the later case, the destination gateway forwards the packets directly to the destination host.

There are in general two ways to perform routing within the induced graph: proactive routing and reactive routing. In [22], DSDV [11] is used as a sample proactive routing to illustrate the dominating-set-based routing. Using the *ns*-2 simulator, Sinha, Sivalumar, and Bharghavan [16] evaluate the performance of DSR [2] and AODV [10] (both are reactive routing), when they are operated over the dominating set (called *core* in [16]) and compare their performance against those of their basic versions.

3. Dominating Set Reduction

In this section, we propose several ways (in form of *rules*) to reduce the size of the connected dominating set derived from the marking process. We first assign a distinct id, $id(v)$, to each vertex v in V'.

Rule 1: Consider two vertices u and v in G'. If $N(u) - \{v\} \subseteq N(v)$ in G and $id(u) < id(v)$, change the marker of u to F; i.e., V' is changed to $V' - \{u\}$.

The above rule indicates that when the neighbor set of u is covered by that of v, vertex u can be removed from V' if the id of u is smaller than that of v. (The

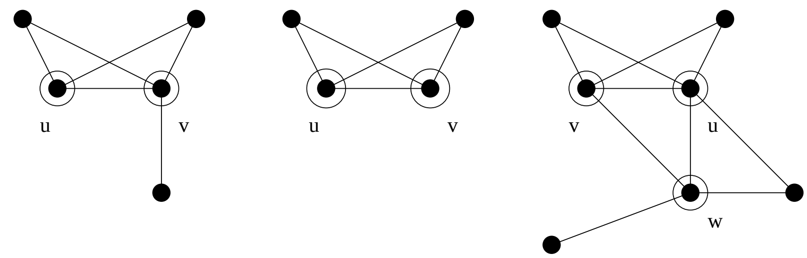

Figure 3: Three examples of dominating set reduction.

importance of the role of id will be discussed later.) When u and v cover each other (i.e., both vertices meet the condition in the rule), the vertex with a smaller id is removed. It is easy to prove that $V' - \{u\}$ is still a connected dominating set of G. In addition, Property 3 still holds. Note that u and v in Rule 1 may or may not be neighbors.

In Figure 3 (a), since $N(u) - \{v\} \subseteq N(v)$, vertex u is removed from V' if $id(u) < id(v)$ and vertex v is the only dominating vertex in the graph. In Figure 3 (b), since u and v cover each other, either u or v can be removed from V'. To ensure one and only one is removed, we pick the one with a smaller id.

Rule 2: Assume that v and w are neighbors in G'. If $N(u) - \{v, w\} \subseteq N(v) \cup N(w)$ in G and $id(u) = \min\{id(u), id(v), id(w)\}$, then change the marker of u to F.

The above rule indicates that when the neighbor set of u is covered by the neighbor sets of two marked vertices, v and w, if u has the smallest id of the three, it can be removed from V'. Again, it is easy to prove that $V' - \{u\}$ is still a connected dominating set. However, Property 3 usually does not hold. Although v and w are directly connected in Rule 2, they may or may not be neighbors of u.

Consider the example in Figure 3 (c) where both v and w are neighbors of u. Clearly, $N(u) - \{v, w\} \subseteq N(v) \cup N(w)$. If $id(u) = \min\{id(u), id(v), id(w)\}$, vertex u can be removed from V' based on Rule 2. If $id(v) < id(u)$ then vertex v can be removed based on Rule 1, since $N(v) - \{u\} \subseteq N(u)$. If $id(w) < id(u) < id(v)$ no vertex can be removed. Therefore, the id assignment also decides the final outcome of the dominating set.

The role of id is very important to avoid "illegal simultaneous" removal of vertices in V' (i.e., vertices are unmarked). In general, vertex u cannot be removed even if $N(u) - \{v\} \subset N(v)$, unless $id(u) < id(v)$. Consider the example in Figure 3 (c) with $id(u) = \min\{id(u), id(v), id(w)\}$. If the above rule were not followed, vertex v would be unmarked to F (because $N(v) - \{u\} \subset N(u)$ even though $id(u) < id(v)$); and based on Rule 2, vertex u would be unmarked to F. Clearly, vertex w in V' does not form a dominating set any more. In the subsequent discussion, we use the alphabetic order of vertex label to order id's. For example, $u < v < w$.

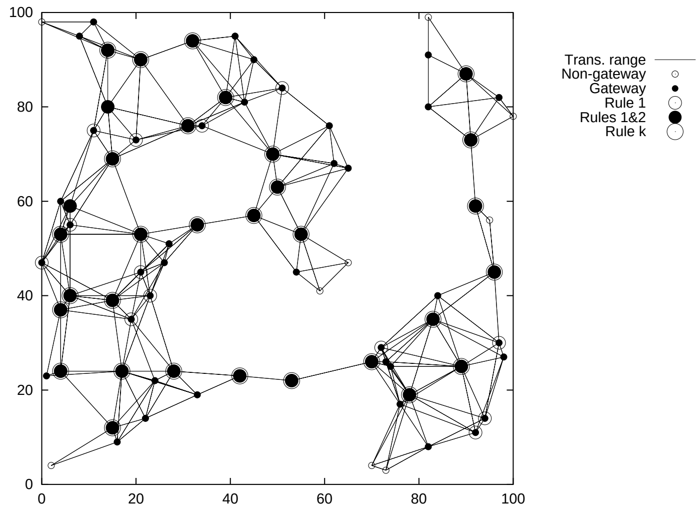

Figure 4: An ad hoc network generated by the simulation software.

If u and v are neighbors in Rule 1, Rule 1 is called *restricted*. Similarly, if u is neighbor of both v and w in Rule 2, Rule 2 is called *restricted*. We will see later that it is relatively easy to implement restricted Rule 1 (Rule 2) in a localized way. To apply restricted Rule 1 or Rule 2, an additional last step in the marking process needs to be included: If u is marked ($m(u) = T$), send its status to all its neighbors; i.e., each host needs to keep 2-hop neighborhood information. To apply non-restricted Rule 1 and Rule 2, u's status needs to be transferred one hop further; i.e., each host needs to keep 3-hop neighborhood information.

Rule 1 and Rule 2 can be easily extended to a more general rule where the neighbor set of vertex u is covered by the union of neighbor sets of more than two vertices in V'. The general rule for the neighbor set of vertex u covered by neighbors sets of k vertices is the following.

Rule k: Assume that $\{v_1, v_2, ..., v_k\}$ is the vertex set of a connected subgraph in G'. If $N(u) - \{v_1, v_2, ..., v_k\} \subseteq N(v_1) \cup N(v_2) \cup ... \cup N(v_k)$ in G and $id(u) = \min\{id(u), id(v_1), id(v_2), ..., id(v_k)\}$, then change the marker of u to F.

Again, the resultant $V' - \{u\}$ is still a connected dominating set after applying Rule k. However, Property 3 usually does not hold. One serious problem in applying Rule k is its high computation cost, even if the restricted Rule k is applied where the computation complexity is choosing k out of $|N(u)|$ neighbors of u. Note that other metrics can be used to break a tie; for example, vertex degree (number of neighbors), energy level, and geographical location of vertex in a particular dimension [19].

Figure 4 shows an ad hoc network generated by the simulation software in a confined space of 100×100. There are 80 hosts each of which has a transmitter range of 20. Non-gateways and gateways derived by the marking process, Rule 1,

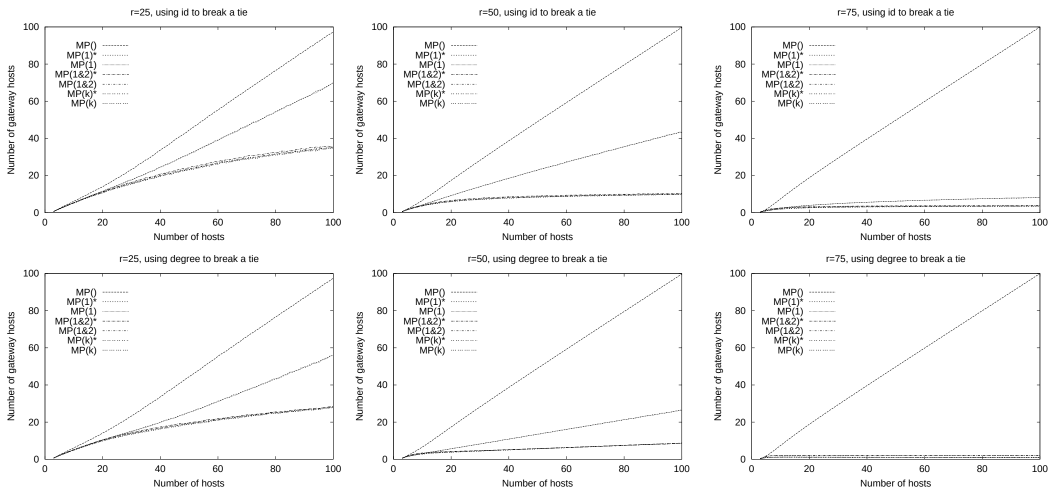

Figure 5: Size of dominating set generated by different versions of the marking process, breaking a tie by vertex id (upper row) or vertex degree (lower row). The transmitter range is 25 (left column), 50 (center column), and 75 (right column).

Rules 1 and 2, and Rule k are represented by cycles of different types. In this example, Rule k unmarks only two more gateways than Rules 1 and 2 do.

Figures 5 and 6 show simulation results on the average size of dominating set generated by $MP()$, $MP(1)$, $MP(1)^*$, $MP(1\&2)$, $MP(1\&2)^*$, $MP(k)$, and $MP(k)^*$. $MP()$ stands for marking process without any rule, $MP(1)$ for marking process with Rule 1, $MP(1)^*$ for restricted $MP(1)$, etc. Two sets of simulation have been conducted. The first one generates connected graphs with a fixed transmitter range (25, 50, or 75). The second one generates connected graphs with a fixed average vertex degree (6, 12, or 18). The marking process and the rules are then applied to the generated connected graphs. With fixed average vertex degree, the number of gateway hosts increases linearly with the number of hosts (see Figure 6). That is the size of a dominating set (derived from the marking process and the reduction methods) relates to vertex degree more than transmitter range. It is clear from the results that $MP(k)$ does not improve much in reducing the number of gateways compared with $MP(1)$ and $MP(1\&2)$, especially in reasonably dense networks. Considering its high computation cost, $MP(k)$ will not be considered in the subsequent discussion.

4. Dominating Set Update

It is assumed that network topology is changed by two operations: host switch-off and host switch-on. Although host movement is not considered in this paper, such an operation can be viewed as several simultaneous and non-simultaneous link connections (switch-on) and disconnections (switch-off). For example, when a host u moves, it may lead to several link disconnections with u's neighbors, and at the same

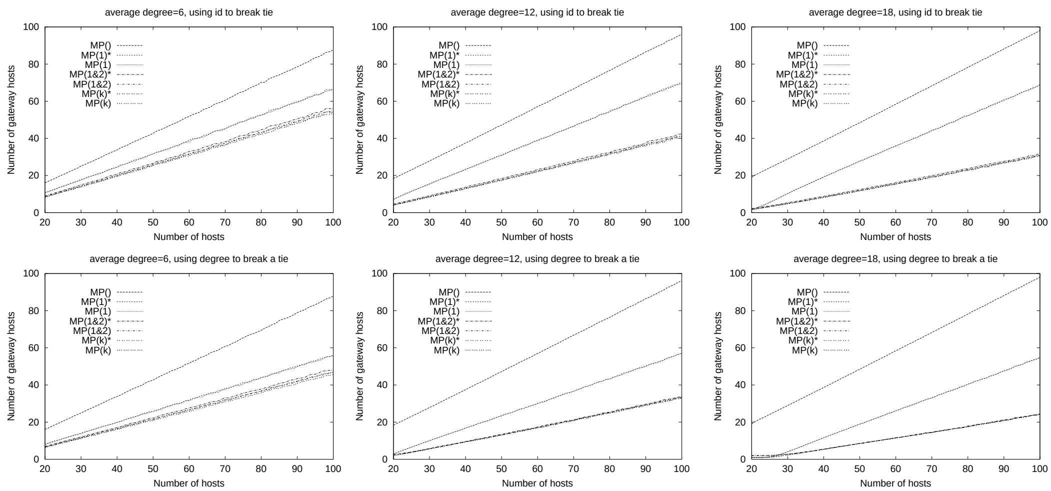

Figure 6: Size of dominating set generated by different versions of the marking process, breaking a tie by vertex id (upper row) or vertex degree (lower row). The average vertex degree is 6 (left column), 12 (center column), and 18 (right column).

time, it may have new link connections to the hosts within its wireless transmitter range. These new links may be disconnected again depending on the way host u moves.

4.1. *Different versions of the marking process*

The update of a dominating set depends on how the dominating set is constructed. In the following, we consider five versions of the marking process:

- Marking process(MP) without Rule 1 and Rule 2: $MP()$.
- MP with restricted Rule 1 only: $MP(1)^*$.
- MP with Rule 1 only: $MP(1)$.
- MP with restricted Rules 1 and 2: $MP(1\&2)^*$.
- MP with Rules 1 and 2: $MP(1\&2)$.

In restricted Rule 1 and/or Rule 2 ($MP(1)^*$ and $MP(1\&2)^*$), it is required that u and v are neighbors in Rule 1 and v and w are neighbors of u in Rule 2. In this case, 2-hop neighborhood information is sufficient in implementing $MP(1)^*$ and $MP(1\&2)^*$. In $MP(1)$ and $MP(1\&2)$, u and v are not necessarily neighbors in Rule 1. Also, v and w are not necessarily neighbors of u in Rule 2. In this way, 3-hop neighborhood information is needed at each host. By default, vertex id is used to break a tie in Rules 1 and 2. If vertex degree is used to break a tie, subscript “deg” is used, such as $MP(1\&2)_{deg}$.

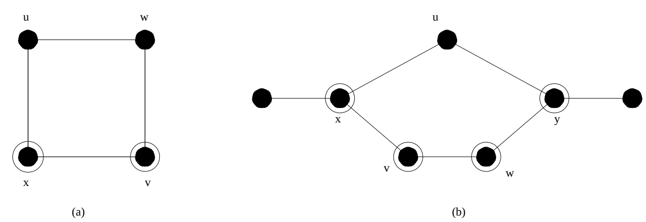

Figure 7: Gateway status after applying (a) Rule 1 and (b) Rule 2.

Consider a graph of four vertices, u, v, w, and x, with four undirected edges (u, w), (v, w), (u, x), and (v, x) as shown in Figure 7 (a). All four vertices are marked using the marking process. Also, $N(u) = N(v) = \{w, x\}$ ($N(w) = N(x) = \{u, v\}$). Using $MP(1)$, one of u and v (also one of w and x) is unmarked (and such a vertex is called *ex-gateway*), leaving two marked vertices (x and v based on Rule 1). Note that ex-gateway hosts are hosts marked by the marking process but unmarked by one of the rules. Using $MP(1)^*$, none of the gateways can be unmarked. Figure 7 (b) shows an example of applying $MP(1\&2)$. Gateway u can be unmarked since its open neighbor set is covered jointly by v's and w's. Note that in this case u is neither a neighbor of v nor a neighbor of w. Note that using $MP(1)^*$ and $MP(1\&2)^*$, gateway u cannot be unmarked.

To simplify the discussion, it is assumed that the marking process (together with Rule 1 and/or Rule 2) can be done quickly between two switch-on/off operations, without requiring each host to apply the marking process at the same time. The period between two switch-on/off operations is called a *phase*. Each host u keeps two statuses: ($m(u)$, $m(u)^*$). $m(u)$ stores the result of the marking process. $m(u)^*$ stores the final result after applying Rule 1 and/or Rule 2. $m(u)^*$ is determined based on $m(v)$ (not $m(v)^*$) of its neighbor v. $m(u)^*$ represents the final status of u, and is independent of the sequence in which hosts in the network apply Rule 1 and/or Rule 2. In the subsequent discussion, we consider the locality of $MP()$, $MP(1\&2)$, and $MP(1\&2)^*$.

4.2. *Update under* $MP()$

The marking process has the following desirable locality property:

Locality property: The status of a host (gateway/non-gateway) depends only on connections of its neighbors, not the status of its neighbors.

The implication of the locality property is that the status of a host is independent of the status of its neighbors. Therefore, when host v switches on/off, hosts and only hosts that are neighbors of v may change their status.

When a mobile host v switches on, only its non-gateway neighbors, along with host v, need to update their status, because any gateway neighbor will still remain as gateway after a new vertex v is added.

Switch-on:

1. Mobile host v broadcasts to its neighbors about its switch-on.
2. Each host $u \in v \cup N(v)$ exchanges its neighbor set $N(u)$ with its neighbors.
3. Host v assigns its marker $m(v)$ to T if there are $(w, v) \in E$ and $(v, w') \in E$, but $(w, w') \notin E$.
4. Each non-gateway neighbor $u \in N(v)$ assigns its marker $m(u)$ to T if there is $(w, u) \in E$, but $(w, v) \notin E$.

The case for a host switch-off is similar to the one for a host switch-on. When a mobile host v switches off, only gateway neighbors of the switch-off host need to update their status, because any non-gateway neighbor will still remain as non-gateway after vertex v is deleted.

Switch-off:

1. Mobile host v broadcasts to its neighbors about its switch-off.
2. Each host $u \in N(v)$ exchanges its neighbor set $N(u)$ with its neighbors.
3. Each gateway neighbor u assigns its marker $m(u)$ to F if all neighbors are pairwise connected; that is, $(w, w^{'}) \in E$ for any two neighbors w and $w^{'}$ of u.

4.3. *Update under* $MP(1\&2)^*$

When the marking process is used together with Rules 1 and 2, the locality property no longer holds: the status of host u depends also on the status of other hosts (v in Rule 1 and v and w in Rule 2). In the restricted Rules 1 and 2, u and v are neighbors in Rules 1 and 2, and u and w are neighbors in Rule 2.

Lemma 1 *When the status of host u changes and u is not a neighbor of any switch-on/off host, then this status change is caused only by using Rule 1 or Rule 2.*

Proof. Since the neighbor set of u remains unchanged, based on the locality property of the marking process, the status of host u cannot be changed by the marking process. □

Theorem 1 shows that status changes are restricted to neighbors of switch-on (switch-off) hosts only. Again, the notion of phase is used during the transition between two phases occurs when several hosts switch on (off).

Theorem 1 *When the dominating set is derived by the marking process with restricted Rules 1 and 2, and in addition, vertex id is used to break a tie in Rules 1 and 2, hosts and only hosts that are neighbors of switch-on (switch-off) hosts need to update their status.*

Proof. Suppose an arbitrarily selected host u is not a neighbor of any switch-on/off host. Based on Lemma 1, u changes its status by neighbor v (v and w) using Rule 1 (Rule 2). Neither v nor w is a switch-on/off host based on restricted Rules 1 and 2 and the way u is selected. The difference between neighbor sets of v and w in the new phase and the previous one is a subset of switch-on/off hosts, with none of them being neighbors of u. We consider the following two cases:

- If host u is changed from non-gateway to gateway, this means that Rule 1 (Rule 2) applied on u in the previous phase cannot be used in the current phase. This occurs when Rule 1 (Rule 2) fails the neighbor coverage condition between u and v in Rule 1 (u, v, and w in Rule 2) in the current phase, which is impossible.

- If host u is changed from gateway to non-gateway, based on neighbor sets of u and v for Rule 1 (u, v and w for Rule 2), Rule 1 (Rule 2) should have been applied to u in the previous phase. This is a contradiction.

□

When a mobile host u switches on, only its non-gateway (including ex-gateway) neighbors, along with host u, need to update their status by the marking process, while any gateway neighbor will remain as gateway. Specifically, non-gateway neighbors may change to gateway neighbors. Using restricted Rules 1 and 2, gateway neighbors may change to ex-gateway neighbors. Ex-gateways in the previous phase that are re-marked by the marking process in the current phase may or may not be unmarked again (back to ex-gateways) by Rules 1 and 2. When a mobile host v switches off, only gateway neighbors (including ex-gateways) of the host need to update their status by the marking process, while any non-gateway (except ex-gateway) neighbor will still remain as non-gateway after vertex v is deleted. Specifically, gateway neighbors may change their status to the non-gateway status. An ex-gateway neighbor may change back to the gateway status by the marking process, by Rules 1 and 2, or it remains ex-gateway. In summary, the same procedures for switch-on (switch-off) discussed in the previous section can be applied. In addition, Rules 1 and 2 need to be applied to all switch-on hosts and neighbors of switch-on (switch-off) hosts.

Note that when a tie in Rules 1 and 2 is broken by vertex degree instead of vertex id, the locality property no longer holds for restricted Rules 1 and 2. When applying Rules 1 and 2, the host with a smaller vertex degree is changed to ex-gateway. In case of a tie, vertex id is used to break it. Consider the example in Figure 8 (a). When vertex degree is used to break a tie in Rule 1, only host u is gateway and

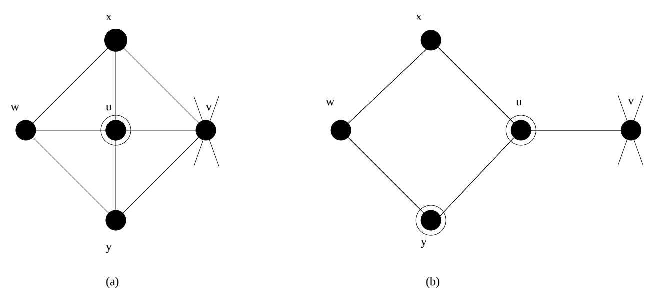

Figure 8: Violation of the locality property in a switch-off operation (on v) when vertex degree is used to break a tie.

hosts v, w, x, and y are ex-gateways. When host v switches off, hosts u and w cover each other with the same vertex degree, w becomes the new gateway (u becomes ex-gateway) when vertex id is used to break a tie. However, w is not a neighbor of the switch-off host v. If vertex id is used to break a tie in Rules 1 and 2, hosts u, w, and y are gateways and host v and x are ex-gateways before host v switches off.

The following result shows that a switch-on/off host will affect the status of its neighbors within 2 hops when the restricted Rule 1 and Rule 2 are applied and vertex degree is used to break a tie.

Theorem 2 *When the dominating set is derived by the marking process with the restricted Rules 1 and 2, and in addition, vertex degree is used to break a tie, hosts and only hosts that are within 2 hops of switch-on (switch-off) hosts need to update their status.*

Proof. Arbitrarily select a host u that is k hops ($k > 2$) away from the switch-on/off host. Based on Lemma 1, the status change of v can only be caused by Rule 1 or Rule 2. Since Rule 1 (Rule 2) is restricted, other hosts v (v and w) used in Rule 1 (Rule 2) must be neighbors of u; i.e., $(k-1)$-hop neighbors of the switch-on/off host. Clearly, vertex degree and vertex id of v and w both remain unchanged in the new phase. Following the similar argument used in the proof of Theorem 1, we conclude that the status of v remains unchanged. □

All results in this subsection also apply to $MP(1)^*$.

4.4. *Update under* $MP(1\&2)$

In $MP(1\&2)$, a switch-on/off host may cause the status change of a host that is 2 hops away as shown in Figure 7 (b).

Theorem 3 *When the dominating set is derived by the marking process with Rules 1 and 2, and in addition, vertex id is used to break a tie, hosts and only hosts that are within 2 hops of switch-on (switch-off) hosts need to update their status.*

Proof. Suppose an arbitrarily selected host u is not a 1-hop or 2-hop neighbor of any switch-on/off host. Based on Lemma 1, u changes its status by neighbor v (v

and w) using Rule 1 (Rule 2). Since the neighbor set of u is covered by the neighbor set of v in Rule 1 (and jointly with the neighbor set of w in Rule 2), both v and w are no more than 2 hops away from u. Therefore, neither v nor w is a switch-on/off host. The difference between neighbor sets of v and w in the new phase and the old one is a subset of switch-on/off hosts, with none of them being neighbors of u. The same arguments used in the proof of Theorem 1 can be applied to show that it is impossible to change the status of host u. □

In a new phase, in addition to the switch-on (switch-off) procedures described in subsection , Rules 1 and 2 need to be applied to all switch-on hosts and 1-hop and 2-hop neighbors of switch-on (switch-off) hosts.

Theorem 3 fails when vertex degree is used to break a tie. Consider the example in Figure 8 (b) where vertex degree is used to break a tie in Rules 1 and 2. Hosts u and y are gateways, w and x are ex-gateways, and v is non-gateway. When host v switches off, hosts u and w cover each other. Host u becomes ex-gateway and w gateway. However, w is 3 hops away from v. On the other hand, the switch-on/off host can only affect the status of neighbors within 3 hops as shown in the following result (its proof is similar to the one for Theorem 3).

Theorem 4 *When the dominating set is derived by the marking process with Rules 1 and 2, and in addition, vertex degree is used to break a tie, hosts and only hosts that are within 3 hops of switch-on (switch-off) hosts need to change their status.*

All results in this subsection also apply to $MP(1)$.

5. Simulation

The simulation software generates random connected ad hoc networks within a confined area of 100×100. Each host in the network is marked as non-gateway, ex-gateway, and gateway by the marking process and the reduction rules (Rules 1 and 2). For each network, one extra host is added (switch-on) in a random position, and the status change of other hosts is computed. In the same manner, one randomly selected host is removed (switch-off), and the status change is computed. Note that Rules 1 and 2 may be implemented in different ways (restricted or non-restricted, breaking a tie by vertex id or vertex degree). In the simulation, networks are generated with a fixed transmitter range or a fixed average vertex degree. When the transmitter range is fixed (25, 50, or 75), the number of hosts ranges from 3 to 100. When the average vertex degree is fixed (6, 12, or 18), the number of hosts ranges from 20 to 100. For each parameter setting, random networks are generated, and dominating set and status change are computed until the confidence intervals of these metrics are acceptable. For the average size of dominating set, the 90% confidence interval is within ±1%. For the average number of status change, the 90% confidence interval is within ±10%.

Figures 9 and 10 show the number of status changes per switch-on/off. Note that when a host switches on (off), only less than one nearby host need to change its

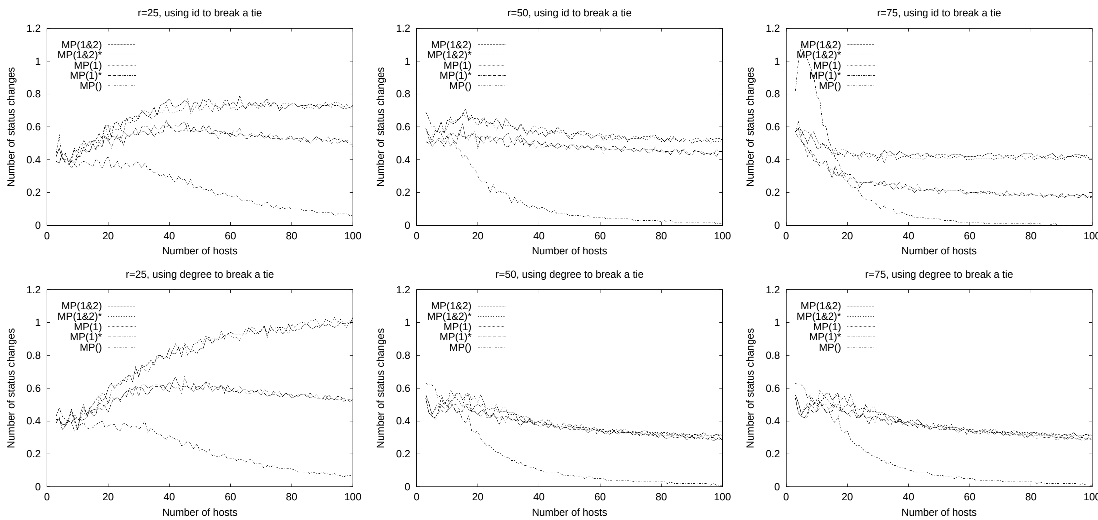

Figure 9: Average number of status changes per switch-on/off, breaking a tie by vertex id (upper row) or vertex degree (lower row). The transmitter range is set to 25 (left column), 50 (center column), and 70 (right column).

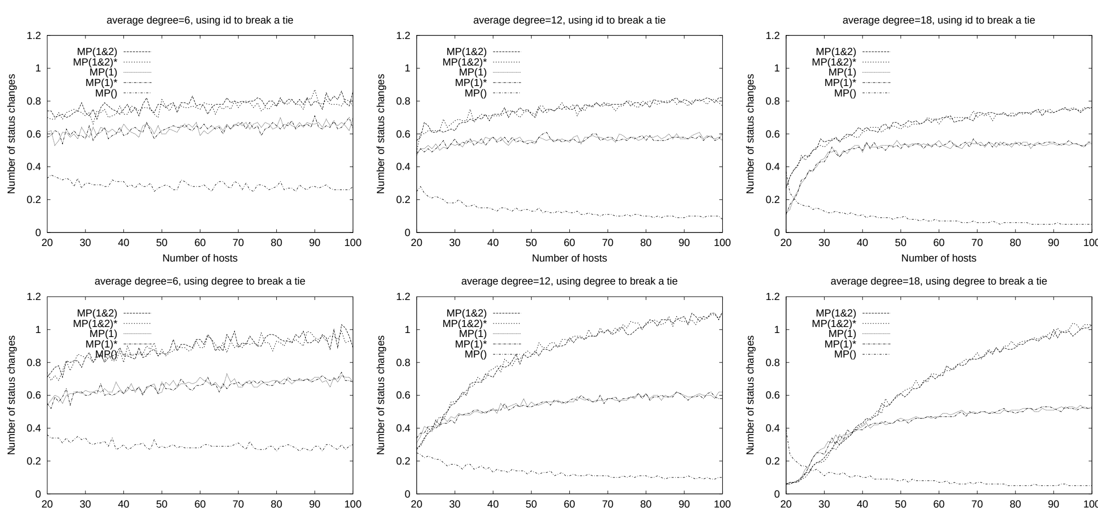

Figure 10: Average number of status changes per switch-on/off, breaking a tie by vertex id (upper row) or vertex degree (lower row). The average vertex degree is set to 6 (left column), 12 (center column), and 18 (right column).

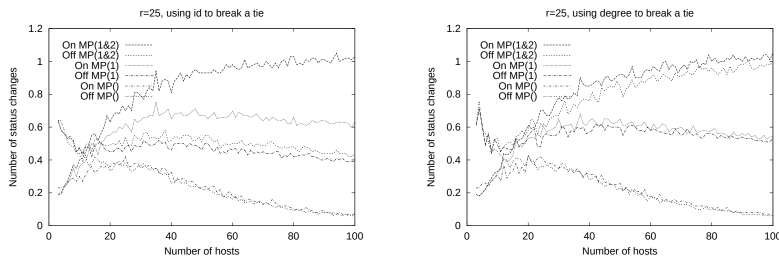

Figure 11: The difference between host switch-on and host switch-off, breaking a tie by vertex id (left column) or vertex degree (right column). The transmitter range is set to 25.

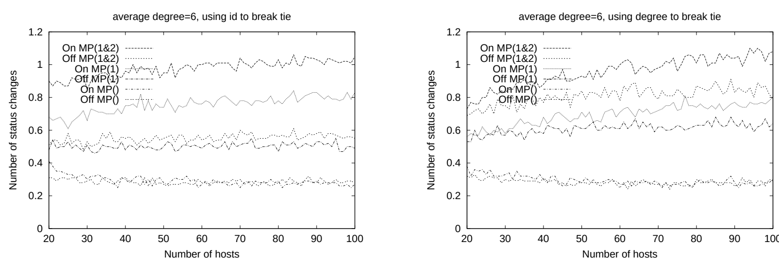

Figure 12: The difference between host switch-on and host switch-off, breaking a tie by vertex id (left column) or vertex degree (right column). The average vertex degree is set to 6.

status. As to different versions of the marking process, $MP()$ is the most stable and $MP(1\&2)$ is the most unstable. The restricted versions ($MP(1)^*$ and $MP(1\&2)^*$) and the non-restricted versions ($MP(1)$ and $MP(1\&2)$) of Rules 1 and 2 are very close in their stability. However, breaking a tie with vertex id is more stable than with vertex degree, unless when the graph is extremely dense.

Figures 11 and 12 compare the difference between host switch-on and host switch-off operations. When only $MP()$ is applied or vertex degree is used to break a tie for $MP(1)$ and $MP(1\&2)$, the number of status changes caused by switch-on and switch-off are very close. However, when vertex id is used to break a tie for $MP(1)$ and $MP(1\&2)$, switch-on causes more status changes than switch-off.

Figures 13 and 14 show that when a host switches on or a non-gateway (ex-gateway) host switches off (the left and center graphs), either using vertex id or using vertex degree to break a tie makes no significant difference to the result. Using vertex degree causes more status changes only when a gateway host switches off (the right graph), which is exactly the situation that causes the difference in the overall stability.

When using vertex degree to break a tie, removing a gateway means that a nearby ex-gateway will become a gateway to cover its neighbors. However, when

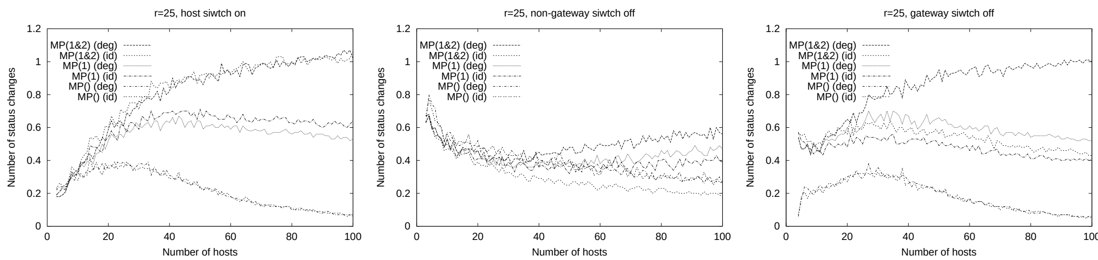

Figure 13: The effect of ways used to break a tie (by vertex id or by vertex degree) when a host switches on (left column), and a non-gateway (center column) or a gateway (right column) switches off. The transmitter range is set to 25.

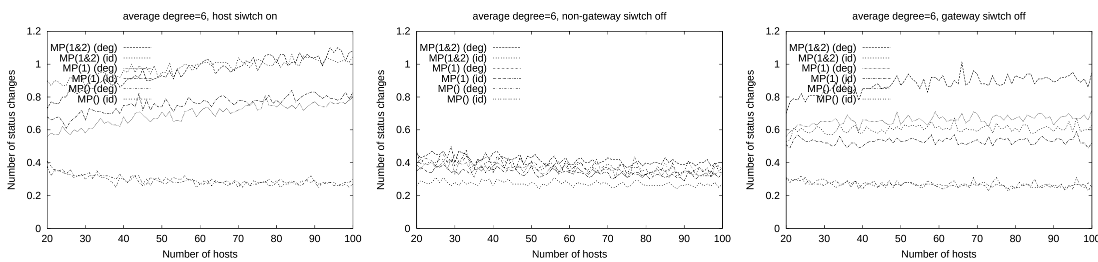

Figure 14: The effect of ways used to break a tie (by vertex id or by vertex degree) when a host switches on (left column), and a non-gateway (center column) or a gateway (right column) switches off. The average vertex degree is set to 6

using vertex id to break a tie, there are more gateways which are already covered by other gateways, and remain marked (instead of unmarked by $MP(1\&2)$) only because they have higher vertex id's. Removing one of these gateways will not cause status change in its neighborhood.

Table 1 shows the average distribution of status changes among the 1-3 hop neighbors of the switch-on/off hosts. The result shows that for all marking processes, any status change can only occur within 3 hops of the switch-on/off host (Total=100%). For $MP()$, any status change is within 1 hop (1-hop=100%), and it is also the case for $MP(1)^*$ and $MP(1\&2)^*$. For $MP(1)$, $MP(1\&2)$, $MP(1)^*_{deg}$, and $MP(1\&2)^*_{deg}$, any status change is within 2 hops (3-hop=0%). Only for $MP(1)_{deg}$ and $MP(1\&2)_{deg}$, some status changes may occur in a 3-hop neighbor. This result verifies the locality properties of different versions of the marking process in Section 3.

Note that although $MP(1)$ and $MP(1\&2)$ may cause status change 1 hop further than $MP(1)^*$ and $MP(1\&2)^*$, the probability of this kind of status change is very low (1.98% when using vertex id and 0.12% when using vertex degree). It is because the average size of the dominating set derived from restricted Rules 1 and 2 is very

Table 1: Locality of different marking processes (r=25).

Marking Process	Change#	1-hop	2-hop	3-hop	Total
$MP()$	0.23	100.00%	0.00%	0.00%	100%
$MP(1)^*$	0.53	100.00%	0.00%	0.00%	100%
$MP(1\&2)^*$	0.66	100.00%	0.00%	0.00%	100%
$MP(1)$	0.53	98.92%	1.08%	0.00%	100%
$MP(1\&2)$	0.67	98.02%	1.98%	0.00%	100%
$MP(1)^*_{deg}$	0.58	94.89%	5.11%	0.00%	100%
$MP(1\&2)^*_{deg}$	0.80	81.07%	19.93%	0.00%	100%
$MP(1)_{deg}$	0.55	93.66%	6.33%	0.01%	100%
$MP(1\&2)_{deg}$	0.80	78.94%	20.94%	0.12%	100%

close to the one of non-restricted Rules 1 and 2 (see Figure 5). On the other hand, it is more expensive to implement $MP(1)$ and $MP(1\&2)$, where 3-hop neighborhood information is needed so that the covered host can obtain the status and neighbor set of the covering hosts. Therefore, restricted Rules 1 and 2 is clearly the choice.

As a conclusion, we can draw the following summary from the simulation results:

1. A host switch-on/off operation only affects the status of its neighborhood within 1 hop ($MP()$, $MP(1)^*$, $MP(1\&2)^*$), 2 hops ($MP(1)$, $MP(1\&2)$, $MP(1)^*_{deg}$, $MP(1\&2)^*_{deg}$), and 3 hops ($MP(1)_{deg}$, $MP(1\&2)_{deg}$).

2. Breaking a tie using vertex id in Rules 1 and 2 is more stable (less status change) than using vertex degree during a switch-on/off operation, especially when a non-gateway host switches off.

3. $MP()$ is more stable than $MP(1)$, which in turn is more stable than $MP(1\&2)$. The restricted and non-restricted versions of a reduction method are very close in stability.

6. Conclusion

In this paper, we have studied the locality property of the dominating set derived from Wu and Li's marking process together with several dominating set reduction methods (Rules 1 and 2). Results show that the marking process has good locality property in a system with switch-on/off hosts. Specifically, only 1-hop neighbors of switch-on/off hosts need to update their gateway/non-gateway status when the restricted Rules 1 and 2 is used. 2-hop neighbors of switch-on/off hosts need to update their gateway/non-gateway status when the non-restricted Rules 1 and 2 is applied. Our results also show that vertex id is better than vertex degree to break a tie situation in terms of stability of dominating set. All these results further confirm that the dominating-set-based routing is a promising approach in ad hoc networks, especially for ones where switch-on/off operations are primary operations that change the network topology.

References

1. L. Blazevic, L. Buttyan, S. Capkun, S. Giordano, J. P. Hubaux, and J. Y. Le Boudec, "Self-organization in mobile ad hoc networks: The approach of terminodes", *IEEE Communications Magazine*, **39**(6) (2001), 166–174.
2. J. Broch, D. Johnson, and D. Maltz, "The dynamic source routing protocol for mobile ad hoc networks", *IETF, Internet Draft, draft-ietf-manet-dsr-00.txt*, 1999.
3. C. C. Chiang, "Routing in clustered multihop, mobile wireless networks with fading channels", *Proc. IEEE SICON*, 1997, pp. 197–211.
4. B. N. Clark, C. J. Colbourn, and D. S. Johnson, "Unit disk graphs", *Discrete Mathematics*, **86** (1990), 165–177.
5. IEEE Standards Departments, *IEEE draft standard - Wireless LAN*, IEEE Press, 1996.
6. D. Estrin, R. Govindan, J. Heidemann, and S. Kumar, "Next century challenges: Scalable coordination in sensor networks", *Proc. ACM MOBICOM*, 1999, pp. 263–270.
7. S. Giordano and W. W. Lu, "Challenges in mobile ad hoc networking", *IEEE Communications Magazine (Guest Editorial)*, **39**(6) (2001) 129–181.
8. S. Guha and S. Khuller, "Approximation algorithms for connected dominating sets", *Algorithmica*, **20**(4) (1998) 374–387.
9. C. R. Lin and M. Gerla, "Adaptive clustering for mobile wireless networks", *IEEE Journal on Selected Areas in Communications*, **15**(7) (1996) 1265–1275.
10. C. E. Perkins, *Ad Hoc Networking*. Addison Wesley, 2001.
11. C. E. Perkins and E. M. Royer, "Highly dynamic destination-sequenced distance-vector routing (DSDV) for mobile computers" *Proc. ACM SIGCOMM*, 1994 pp. 234–244.
12. "Ricochet networks", *http://www.ricochet.com/*, 2002.
13. R. Ruppe, S. Griswald, P. Walsh, and R. Martin, "Near term digital radio (NTDR) system", *Proc. IEEE MILCOM*, .
14. T. Shepard, "A channel access scheme for large dense packet radio networks", *Proc. ACM SIGCOMM*, 1996, pp. 219–230.
15. A. Sinha and A. Chandrakasan, "Dynamic power management in wireless sensor networks", *IEEE Design & Test of Computers*, **18**(2) (2001) 62–74.
16. P. Sinha, R. Sivakumar, and V. Bharghavan, "Enhancing ad hoc routing with dynamic virtual infrastructures", *Proc. IEEE INFOCOM*, 2001, pp. 1763–1772.
17. R. Sivakumar, B. Das, and V. Bharghavan, "An improved spine-based infrastructure for routing in ad hoc networks", *Proc. 3rd IEEE Symp. Computers and Communications*, 1998.
18. I. Stojmenovic and X. Lin, "Power-aware localized routing in wireless networks" *IEEE Transactions on Parallel and Distributed Systems*, **12**(11) (2001), 1122–1133.
19. I. Stojmenovic, M. Seddigh, and J. Zunic, "Internal node based broadcasting algorithms in wireless networks", *Proc. 34th Annual IEEE Hawaii Int'l Conf. System Sciences*, 2001.
20. C.-K. Toh, "Maximum battery life routing to support ubiquitous mobile computing in wireless ad hoc networks", *IEEE Communications Magazine*, **39**(6) (2001), 138–147.
21. J. Wu, "Extended dominating-set-based routing in ad hoc wireless networks with unidirectional links", *IEEE Transactions on Parallel and Distributed Systems*, **13**(9) (2002), 866–881.
22. J. Wu and H. Li, "On calculating connected dominating sets for efficient routing in ad hoc wireless networks", *Proc. 3rd Int'l Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications*, 1999, pp. 7–14.