Comment

Comments on "A Systematic (16, 8) Code for Correcting Double Errors and Detecting Triple-Adjacent Errors"

Rajendra S. Katti

***Abstract*—In this paper we first present a systematic (16, 8) code that can correct double errors and detect all triple-adjacent errors. The restriction of detecting triple-adjacent errors in 8-bit bytes has been removed. We also present a systematic (16, 8) code that can correct all 16 single errors, correct 113 of the 120 double errors, detect seven double errors, correct seven of 14 triple adjacent errors, detect seven triple adjacent errors, and correct all quadruple adjacent errors.**

***Index Terms*—Adjacent error detection, error correction, systematic codes.**

I. Introduction

In [1] a systematic (16, 8) code was presented for correcting double errors and detecting triple-adjacent errors within 8-bit bytes. We will first rearrange the columns of the parity matrix H given in [1] to obtain a new matrix H that can correct double errors and detect all triple-adjacent errors. The restriction of detecting triple-adjacent errors within 8-bit bytes has been removed. We will then give another parity matrix H that results in a systematic (16, 8) code that can correct all 16 single errors, correct 113 of the 120 double errors, detect seven double errors, correct seven of 14 triple adjacent errors, detect seven triple adjacent errors and correct all quadruple adjacent errors. In the next section we give the two H matrices and demonstrate their error detection/correction capabilities.

II. Code Construction

We start with the H matrix of the systematic (16, 8) code in [1] that can correct all double errors and detect triple-adjacent errors in 8-bit bytes. This matrix is,

$$H = \begin{bmatrix} 0&0&1&0&1&0&1&1&1&0&0&0&0&0&0&0 \\ 1&0&0&1&0&1&0&1&0&1&0&0&0&0&0&0 \\ 1&1&0&0&1&0&1&0&0&0&1&0&0&0&0&0 \\ 0&1&1&0&0&1&0&1&0&0&0&1&0&0&0&0 \\ 1&0&1&1&0&0&1&0&0&0&0&0&1&0&0&0 \\ 0&1&0&1&1&0&0&1&0&0&0&0&0&1&0&0 \\ 1&0&1&0&1&1&0&0&0&0&0&0&0&0&1&0 \\ 0&1&0&1&0&1&1&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

Note that the first eight columns of H form an 8×8 circulant submatrix C, and the last eight columns form an 8×8 identity matrix. The above matrix can detect triple-adjacent errors in 8-bit bytes as the syndromes for the triple-adjacent errors at the 8-bit boundary is the same as the syndrome for a double error. For example the sum (modulo 2) of the eighth, ninth, and tenth columns of H is,

$$S = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

The above syndrome is the same as the syndrome for a double error (sum of the twelveth and fourteenth column). We shall now rearrange the columns of H to avoid this problem and hence the resulting code can correct all double errors and detect all triple-adjacent errors. The new H matrix is,

$$H = \begin{bmatrix} 0&0&1&0&1&0&1&1&1&0&0&0&0&0&0&0 \\ 1&0&0&1&0&1&0&1&0&0&0&0&0&0&0&1 \\ 1&1&0&0&1&0&1&0&0&0&0&0&0&0&1&0 \\ 0&1&1&0&0&1&0&1&0&0&0&0&0&1&0&0 \\ 1&0&1&1&0&0&1&0&0&0&0&0&1&0&0&0 \\ 0&1&0&1&1&0&0&1&0&0&0&1&0&0&0&0 \\ 1&0&1&0&1&1&0&0&0&0&1&0&0&0&0&0 \\ 0&1&0&1&0&1&1&0&0&1&0&0&0&0&0&0 \end{bmatrix}$$

Note that the columns of the circulant submatrix C have not been rearranged. However the columns of the identity submatrix have been rearranged. The two syndromes for triple-adjacent errors that stradle the 8-bit boundary are, (1 1 1 1 1 1 0 1)^T and (0 1 0 1 0 1 0 1)^T (the T denotes transpose). The first syndrome above has weight 7 and this does not result in any problems as the syndromes for single and double errors have weights less than seven. The second syndrome above also results in no problems as all weight 4 syndromes for single and double errors are circular shifts of the following syndromes, (1 1 1 1 0 0 0 0)^T, (0 0 1 0 0 1 1 1)^T, (1 1 0 0 1 1 0 0)^T, (0 1 1 0 1 0 1 0)^T. The other syndromes resulting from triple-adjacent errors are all circular shifts of the following syndromes, (1 1 1 0 0 0 0 0)^T and (1 1 0 0 0 1 0 1)^T. The above two syndromes do not coincide with any single or double error syndrome as the other weight 3 syndromes due to single or double errors are all circular shifts of the following syndrome, (0 0 1 0 1 0 1 0)^T. All the syndromes are now stated again for clarity. The syndromes due to single errors are all circular shifts of the following syndromes, (1 0 0 0 0 0 0 0)^T and (0 1 1 0 1 0 1 0)^T. The syndromes due to double errors are all circular shifts of the following syndromes, (1 1 0 0 0 0 0 0)^T, (1 1 1 0 1 0 1 0)^T, (0 0 1 0 1 0 1 0)^T, (0 1 0 1 1 1 1 1)^T, (1 1 1 1 0 0 0 0)^T, (0 0 1 0 0 1 1 1)^T, (1 1 0 0 1 1 0 0)^T. The syndromes due to triple-adjacent errors are all circular shifts of the following syndromes, (1 1 1 0 0 0 0 0)^T, (1 1 0 0 0 1 0 1)^T and the following syndromes are a result of triple-adjacent errors stradled at the 8-bit boundary, (1 1 1 1 1 1 0 1)^T, (0 1 0 1 0 1 0 1)^T. It should be noted that not only can the triple-adjacent errors be detected they can also be corrected.

We now give another H matrix that results in a systematic (16, 8) code that can correct all 16 single errors, correct 113 of the 120 double errors, detect seven double errors, correct seven of 14 triple adjacent errors, detect seven triple adjacent errors, and correct all quadruple adjacent errors. The new H matrix is formed by alternating the columns of the circulant submatrix C and the identity submatrix of the original H in [1]. This matrix is,

Manuscript received Nov. 8, 1994.

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$$H = \begin{bmatrix} 0&1&0&0&1&0&0&0&1&0&0&0&1&0&1&0 \\ 1&0&0&1&0&0&1&0&0&0&1&0&0&0&1&0 \\ 1&0&1&0&0&1&0&0&1&0&0&0&1&0&0&0 \\ 0&0&1&0&1&0&0&1&0&0&1&0&0&0&1&0 \\ 1&0&0&0&1&0&1&0&0&1&0&0&1&0&0&0 \\ 0&0&1&0&0&0&1&0&1&0&0&1&0&0&1&0 \\ 1&0&0&0&1&0&0&0&1&0&1&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&0&1&0&1&0&0&1 \end{bmatrix}$$

In this case the syndromes for single and double errors are the same as the previous H matrix. The syndromes for triple-adjacent errors are seven circular shifts of the following syndromes, $(1\ 1\ 0\ 1\ 1\ 1\ 1\ 1)^T$, $(1\ 1\ 1\ 1\ 0\ 1\ 0\ 1)^T$. Since the syndrome $(1\ 1\ 1\ 1\ 0\ 1\ 0\ 1)^T$ coincides with one of the syndromes for double errors, seven of the double errors and seven of the triple-adjacent errors can only be detected. The syndromes for quadruple-adjacent errors are seven circular shifts of the following syndromes, $(1\ 0\ 0\ 1\ 1\ 1\ 1\ 1)^T$, $(0\ 1\ 1\ 0\ 1\ 1\ 1\ 1)^T$. These syndromes do not coincide with any of the other syndromes and the code can therefore correct all quadruple-adjacent errors.

REFERENCES

[1] T.A. Gulliver and V.K. Bhargava, "A systematic (16, 8) code for correcting double errors and detecting triple-adjacent errors," *IEEE Trans. Computers*, vol. 42, no. 1, pp. 109-112, Jan. 1993.

Authors' Reply

T. Aaron Gulliver and Vijay K. Bhargava

***Abstract*—A (16, 8) double error correcting code is presented which can also correct all triple and quadruple adjacent errors, and detect all errors in 5 or 6 adjacent bits. This code is a modification of a quasi-cyclic code presented previously.**

***Index Terms*—Adjacent errors, random errors, error detection and correction.**

In [1] a (16, 8) double error correcting quasi-cyclic (QC) code was presented which can also detect all triple-adjacent errors within each 8-bit byte. This code is useful in computer memory applications where adjacent errors are more likely than single random errors. Two rearrangements of the parity check matrix of this code are given in [2]. The first allows for the correction of all triple adjacent errors, in addition to double error correction. The second can correct most double and triple adjacent errors, and also all quadruple adjacent errors.

A new arrangement of the parity check matrix is presented here which preserves double error correction, and also allows for triple and quadruple adjacent error correction. In addition, errors in 5 and 6 adjacent bits can be detected.

The (16, 8) systematic QC code in [1] has a generator matrix of the form

$$G = [I_8; C] \tag{1}$$

where I_8 is an 8×8 identity matrix and C is an 8×8 binary circulant matrix. The parity check matrix of this code is

$$H = [C^T; I_8] \tag{2}$$

which is an eight row by 16 column matrix.

The syndromes are the 8-bit vectors $s = eH$ which are just a linear sum of columns of H. The syndromes for all correctable error patterns s_c, must be distinct and nonzero, and, in addition, the syndrome of a detectable error pattern must not be equal to any s_c. Thus to determine the error correction and detection capabilities of a code, a comparison must be made of all relevant syndromes.

There are $2^{n-k} = 2^8$ 8-bit syndromes. Of these, $1 + 16 + 120 = 137$ are used by the zero and correctable error patterns. Thus the triple and quadruple adjacent error patterns must have distinct syndromes contained in the remaining 119. The structure of the QC code can be exploited to reduce the search required to enumerate the syndromes. The generator matrix of this code is

$$G = \left[\begin{array}{cccccccc|cccccccc} 1&0&0&0&0&0&0&0&0&1&1&0&1&0&1&0 \\ 0&1&0&0&0&0&0&0&0&0&1&1&0&1&0&1 \\ 0&0&1&0&0&0&0&0&1&0&0&1&1&0&1&0 \\ 0&0&0&1&0&0&0&0&0&1&0&0&1&1&0&1 \\ 0&0&0&0&1&0&0&0&1&0&1&0&0&1&1&0 \\ 0&0&0&0&0&1&0&0&0&1&0&1&0&0&1&1 \\ 0&0&0&0&0&0&1&0&1&0&1&0&1&0&0&1 \\ 0&0&0&0&0&0&0&1&1&1&0&1&0&1&0&0 \end{array}\right]$$

and H is given by

$$H = \left[\begin{array}{cccccccc|cccccccc} 0&0&1&0&1&0&1&1&1&0&0&0&0&0&0&0 \\ 1&0&0&1&0&1&0&1&0&1&0&0&0&0&0&0 \\ 1&1&0&0&1&0&1&0&0&0&1&0&0&0&0&0 \\ 0&1&1&0&0&1&0&1&0&0&0&1&0&0&0&0 \\ 1&0&1&1&0&0&1&0&0&0&0&0&1&0&0&0 \\ 0&1&0&1&1&0&0&1&0&0&0&0&0&1&0&0 \\ 1&0&1&0&1&1&0&0&0&0&0&0&0&0&1&0 \\ 0&1&0&1&0&1&1&0&0&0&0&0&0&0&0&1 \end{array}\right]$$

Now, rearrange the columns of H to the following

$$H' = \begin{bmatrix} 0&0&0&0&1&0&0&0&1&0&0&1&1&0&1&0 \\ 1&0&0&0&0&0&1&0&0&0&1&0&0&1&1&0 \\ 1&0&1&0&0&0&0&0&1&0&0&0&1&0&0&1 \\ 0&1&1&0&1&0&0&0&0&0&1&0&0&0&1&0 \\ 1&0&0&1&1&0&1&0&0&0&0&0&1&0&0&0 \\ 0&0&1&0&0&1&1&0&1&0&0&0&0&0&1&0 \\ 1&0&0&0&1&0&0&1&1&0&1&0&0&0&0&0 \\ 0&1&1&0&0&0&1&0&0&1&1&0&1&0&0&0 \end{bmatrix}$$

which is just the columns from C and I interleaved, starting with the first column in C and the fourth column in I. Because the columns of H have not been altered, the syndromes for single and double errors remain the same. These are cyclic shifts of columns a to i in Table I, which lists the coefficients of $s(x)$ with the lowest degree at the top. The syndrome weights are given to aid in distinguishing between them. The syndromes for triple-adjacent errors are given in columns j and k. Column j has weight 5 and differs from the only other column

Manuscript received Dec. 7, 1994; revised Jan. 12, 1995.

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